Security-Constrained Design of Isolated Multi-Energy Microgrids

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Abstract—Energy supply in rural and off-grid communities has traditionally relied on diesel-based microgrids, due to limited access. But global environmental concerns are pushing for the transformation of these systems into renewable-based microgrids. This transition to more complex systems with a mix of dispatchable and non-dispatchable resources requires new planning tools that ensure the security of supply. This paper presents a novel mixed-integer linear optimization model that determines optimal technology mix, size, placement, and associated dispatch for a multi-energy microgrid. The model satisfies microgrid’s electrical and heat transfer network limitations by integrating linear power flow and heat transfer equations. It captures the efficiency gains from waste heat recovery through combined heat and power technologies, by modeling the interplay between electrical and heat sources. To ensure a secure design against generator outages, the optimization maintains sufficient reserve capacity in the system, which is dynamically allocated based on system operating conditions. Several case studies on an isolated microgrid in Alaska, illustrate how the proposed model works. The results show the effectiveness of the model and are used to discuss various aspects of the optimization solution.

Index Terms—microgrid, isolated, remote, N-1 contingency, security-constrained, optimal planning, optimal dispatch, mixed integer linear program, MILP

I. NOMENCLATURE

We denote variables in italic fonts, parameters in non-italic fonts, and binary/integer variables with all-small letters. The nomenclature is sorted alphabetically.

Sets and indices

- n, n’ microgrid nodes: 1, 2,…, N
- s storage technologies: electric storage (ES), heat storage (HS), cold storage (CS)
- t time
- u energy use: electricity (EL), cooling (CL), heating (HT)

Parameters

- α j useful heat recovered from a unit of generated electricity
- η j electrical loss coefficient for heat transfer pipe (n,n’), % per meter
- η j charging efficiency of generation technology j
- η s discharging efficiency of storage technology s
- ψ s losses due to self-discharge in storage technology s, % per Δ t
- φ generation or load power factor
- Δ t optimization time-step, hour
- Δ t,cfg post-contingency dispatch period duration, seconds
- Δ t,ramp post-contingency ramp-up period duration, seconds
- AR i annuity rate for technology i
- C C , n Curtailment cost (post-contingency) for electrical loads at node n, $/kW
- CER j carbon emissions rate from generation technology j, kg/kWh
- C P a coefficient of performance for absorption chiller
- C P e coefficient of performance for electric chiller
- C R , s maximum charge rate of storage technology s, % of capacity
- D R , s maximum discharge rate of storage technology s, % of capacity
- E s minimum acceptable energy (state of charge) for storage technology s, %
- E s maximum acceptable energy (state of charge) for storage technology s, %
- F C , k fixed capital cost of continuous technology k, $
- G C j generation cost (e.g. fuel consumption) of technology j, $/kWh
- H n,n’ heat transfer capacity for pipe (n,n’), kW
- M an arbitrary large number
II. INTRODUCTION

The attention towards microgrids is increasing at a fast pace, due to their benefits in terms of renewable integration, low carbon footprint, reliability and resiliency, power quality, and economics. However, microgrids have been the only solution for rural and off-grid communities for a long time, due to the limited/lack-of access to the main grid [1]. Traditionally, these off-grid communities relied on diesel generation to supply their loads, despite the higher fuel prices in the remote areas, but nowadays with increased global environmental concerns and incentives for a transformation of these diesel-based systems into renewable-based microgrids, changes can be observed [2]. This transition to resources with high variability and uncertainty, such as wind and photovoltaic, requires new microgrid planning tools to ensure the security of supply of these isolated systems.

A comprehensive microgrid investment and planning optimization must address (a) power generation mix selection; (b) resource sizing and allocation; (c) operation scheduling [3]; and (d) interplay between electricity, cooling, and heating loops in the microgrid to take full advantage of excess heat. (e) Moreover, in the context of remote/isolated microgrids, accounting for security of supply constraints in the design and operation is needed. A review of the literature (comprehensive reviews of many of the existing tools and computer models for renewable energy integration and microgrid planning can be found in [3]–[5]) shows that most of the existing models focus on individual sub-problems and do not include the others, or include them without enough depth.

Several examples of microgrid design formulations that only tackle the electrical energy flow, neglecting heating and cooling, are given in [6]–[9]. Among this category are also some of the distribution network planning formulations [10]–[13] that consider distributed energy resources (DER), since they share some of the same characteristics with the microgrid design problem. These methods only model electrical energy use and usually consider a limited generation mix.

Among the models that account for different energy uses are [14]–[18]. Omu et al. [14] formulated a mixed integer linear program for the technology selection, unit sizing, unit allocation, and distribution network structure of a distributed energy system that meets the electricity and heating demands of a cluster of buildings. This work, however, models electrical energy as a commodity whose transfer from one location to another is decided without physical laws, i.e. power flow constraints or Kirchhoff laws. Similarly, [15]–[17] present approaches for design and planning of urban and distributed energy systems, but do not include power flow equations. Basu et al. use power loss sensitivity to guide the optimization in... 
siting Combined Heat and Power (CHP)-based DERs in [18]. Although both electrical and thermal networks are modeled, the formulation is nonlinear, and solves using a stochastic approach, which entails a significant computational burden. Furthermore, obtaining an optimal solution is not scalable in such solution methods.

The existing literature also includes references that consider security of supply in the microgrid design. In the literature on grid-connected systems, some methods focus on improving the reliability and security of supply in the distribution system, through leveraging the design of multiple distribution system connected microgrids [19]–[21]. Alternatively, in the context of an individual microgrid, some references integrate security of supply indices, e.g., Loss of Load Expectation (LOLE), as constraints into the problem of optimal sizing and placement of DERs in the microgrid. However, due to the probabilistic nature of the indices, these methods result in nonlinear stochastic formulations [22] that require complex solution methodologies, e.g., meta-heuristics combined with Monte Carlo simulations [23] or Robust Optimization combined with Benders decomposition [7]. Although these methods can capture the uncertainty associated with the security of supply, they cannot be applied to complex multi-energy microgrids due to two main reasons: First, they mostly fail to capture the interplay between electricity, heating, and cooling loads and sources. Second, these techniques entail significant computational burdens and cannot guarantee a certain degree of optimality, especially when applied to large problems such as multi-energy microgrids.

To address the gap in the literature, this paper aims at including the security of supply constraint in the optimal design of isolated multi-energy microgrids and proposes a novel model for N-1 security-constrained design of such systems. The proposed model builds on the Distributed Energy Resources Customer Adoption Model (DER-CAM) developed by Lawrence Berkeley National Laboratory [24],[25]. The contributions of this work are threefold:

- First, we propose an integrated design approach that determines the optimal mix, size, location, and dispatch of renewable and fossil fuel-based DERs in multi-energy microgrids with electricity, heating, and cooling energy uses. To meet the electrical and thermal network constraints, we integrate linear power flow (LinDistFlow) and heat transfer equations into this formulation.

- Second, we integrate a set of linear constraints into the optimization problem that ensure security of supply against N-1 generator contingencies. The constraints are developed such that the optimization run time remains tractable.

- And third, we apply the proposed formulation to an example isolated microgrid developed based on a real isolated microgrid in Alaska.

This paper is organized as follows. Section III introduces the mathematical model for integrated design of multi-energy microgrids, i.e., a microgrid with electrical, heating, and cooling loads, which includes cabling (electrical) and piping (heating) networks. Section IV presents our proposed model for the security-constrained design and operation of the microgrid. In section V, the example case of an isolated utility microgrid in Alaska is studied and discussed. Finally, conclusions and future work are presented in section VI.

III. MATHEMATICAL MODEL FOR INTEGRATED DESIGN OF MULTI-ENERGY MICROGRIDS

The goal is to develop a model that determines the optimal mix, capacity, and siting (placement) of various DER technologies that minimize (a) the overall investment and operation cost and/or (b) the overall CO₂ emissions, while ensuring security of the supply. Since the optimization model becomes very large and due to the superiority of Mixed Integer Linear Program (MILP) solvers over nonlinear ones, we formulate the problem as a MILP, by making necessary simplifying assumptions, to keep the problem solvable in reasonable and practical run times.

We consider a generic microgrid structure that has a radial electrical network and an arbitrary piping network. The load at each node is composed of electricity (plug loads), heating (space- and water-heating), and cooling (space-cooling and refrigeration) end-uses. Recognizing these three end-uses enables the formulation to optimally meet the loads by leveraging the synergies between different energy carriers, since each end-use can be met by multiple technologies. For instance:

- a combination of conventional generators, renewable resources, and battery technologies can be used to supply electrical end-uses;
- cooling loads can be met by electrical chillers, absorption chillers, or cold storage technologies; and
- heating loads can be met by gas-fired boilers, recovered heat from CHP technologies, and heat storage technologies.

A visual representation of our energy conversion model is depicted in Fig. 1 (similar examples can be found in [26], [27]), showing sources and sinks of electricity, heating, and cooling, at each of the microgrid nodes. In this figure, different forms of energy are shown with different arrow heads to enhance readability and also emphasize on the energy conversion. For instance, waste heat from CHP technologies (e.g., ICE, MT, or FC) is recovered by heat exchangers (HX). Other examples are conversion of electrical and heating into cooling by electric (EC) and absorption chillers (AC), respectively.

A. Continuous vs. discrete investment decision variables

We model capacity of DER technologies using a continuous or discrete variable: If a technology is available in small enough modules (e.g., photovoltaic and storage), the optimal capacity is modeled as a continuous variable, significantly lowering the computation time. These technologies are referred to as continuous technologies. Discrete variables are used otherwise (e.g., micro-turbines) and called discrete technologies. Whenever possible, we use continuous variables to model the installed capacity of a given technology, as this significantly contributes to reducing the model runtime.

B. Time resolution

We propose to use typical day-types to model a full year. More specifically, we define a typical “week” day, “weekend” day, and “peak” (e.g. outlier in terms of larger load or larger ramp rate) day per month, where each one is modeled with
representative hourly load profiles. As a result, our proposed formulation models a year with 12x3x24 time-steps, which is less than one-tenth of the number of time-steps in the more common 8,760-hour modeling. This gain is obtained without losing any valuable information in the load profiles, since the impact of peak days on component sizing is captured through inclusion of “peak” days; and the operation cost is mostly determined by typical “week” and “weekend” day-types due to their higher frequency of occurrence. It is worth noting that the formulation can be generalized to include multiple “peak” day types, e.g., one “peak” day profile per end-use or energy carrier.

\[
\begin{align*}
C_{\text{Cost}} &= \sum_{n,g} n_{l,n,g} \cdot P_{l,n,g} \cdot TCC_{l,g} \cdot AR_{l,g} \\
&+ \sum_{n,k} (FCC_k \cdot b_{l,n,k} + VCC_k \cdot C_{n,k}) \cdot AR_k \\
&+ \sum_{n,t} P_{g,n,t} \cdot GC_j \\
&+ \sum_{n,t} p_{l,cur} \cdot CC_n \\
C_{\text{CO2}} &= \sum_{n,t} p_{g,n,t} \cdot CER_j
\end{align*}
\]

It is worth noting that the operation costs in (1) are scaled up from the 864 time-steps to 8,760 time-steps by considering the number of day-types per month, e.g., 20 “week” days, 8 “weekend” days, and 2 “peak” days. The scaling is now shown in (1) to simplify the presentation of the equation.

We adopted LinDistFlow, a distribution-level tractable linear balanced AC power flow model [28], [29]. This model (3)-(6) is advantageous over the well-known DC power flow approximation, since it allows for voltage magnitude deviations in the network, considers line resistances, and models both active and reactive power flow in the network. The net injected active power at a node, \(P_{in,t}\), accounts for DER generation, electric load, electric chiller consumption, and electric storage system charging/discharging (7). Equations (8)-(9) enforce bus voltage constraints and line power constraints in the network, respectively. To linearize line power capacity constraints, we use an inner approximation of the exact constraint, i.e. the octagon in Fig. 2, instead of the circle, using the constraints in (9).

\[
V_{S_{n,t}} - V_{S_{n',t}} = 2 \cdot (R_{n,n'} \cdot P_{n,n'} + X_{n,n'} \cdot Q_{n,n'}) \\
V_{S_{n,t}} = V_0 \\
P_{in,t} = \sum_{n'} P_{n,n'} \\
Q_{in,t} = \sum_{n'} Q_{n,n'} \\
S_{b} \cdot P_{in,t} = \sum_{\{PV,ICE,MT,FC\}} p_{g,n,t} \cdot P_{n,u=EL,t} \\
- \frac{1}{C_{pe}} \cdot p_{g,n,c=EC,t} \\
+ DR_{n,s=ES,t} \cdot \eta_{s=ES} \cdot \frac{1}{\eta_{s=ES}} \cdot C_{R,n,s=ES,t} \\
V^2 \leq V_{S_{n,t}} \leq V^2 \\
\pm Q_{n,n'} = \cotan \left( \frac{1}{2} - e \right) \cdot \left( P_{n,n'} - \cos \left( e \cdot \frac{\pi}{4} \right) \cdot \tan \left( e \cdot \frac{\pi}{4} \right) \right)
\]

\( e \in \{1, ..., 4 \} \)

---

**C. MILP optimization model**

The objective is to minimize the overall microgrid investment and operation cost (1) or its CO2 emissions (2), or a combination of the two objectives. The overall cost (1) includes annualized investment costs of technologies, where annuity rate depends on the interest rate and technology lifetime; generation cost for electrical, heating, or cooling technologies; and cost of post-contingency load curtailments. The emission objective (2) captures CO2 emissions from the operation of all technologies.
The heat balance equation at each node (10) accounts for heat generation; recovered CHP heat; heating loads inclusive of the required heat for absorption chilling, heat from/to storage technologies; and heat transfer between nodes through the piping network considering losses. Equation (11) enforces the pipe capacities. The cooling load at each node can be met by a combination of electric and absorption chilling and energy from cold storage technology (12).

\[
0 = \sum_{g \in \{ST, BL\}} P_{g,n,t} + \sum_{g \in \{ICE, MV\}} a_g \cdot P_{g,n,t} - P_{l,n=HT,t} \frac{1}{\eta_{HS}} \cdot CR_{n,n'=HS,t} + \sum_{g \in \{AC\}} P_{g,n,t} - \sum_{n'} H_{n,n',t} + \sum_{n'} (1 - \gamma_{n,n'}) \cdot H_{n,n',t} \\
0 \leq H_{n,n',t} \leq \bar{H}_{n,n'}
\]

\[
0 = \sum_{c \in \{AC, EC\}} P_{g,n,c,t} - P_{l,n=C,L,t} + \eta_{c=CS} \cdot \frac{1}{\eta_{c=CS}} \cdot CR_{n,n'=CS,t} \\
\]

The literature on district heating networks includes a wide spectrum of modeling approaches, ranging from simple linear models with linear heat loss equations [17] to complex nonlinear models that include details such as network heat and pressure loss, temperature dynamics, etc. [30]. In this work, we use the former approach, in order to preserve optimization model linearity.

The energy (E) in electrical, heat, and cold storage technologies, considering self-discharge, are tracked (13) and kept within limits (14). The rate of charging (CR) and discharging (DR) is also limited.

\[
E_{n,t} = (1 - \varphi_s) \cdot E_{n,t-1} + CR_{n,n'=t} \cdot \Delta t - DR_{n,n'=t} \cdot \Delta t \\
E_s \leq E_{n,t} \leq \bar{E}_s
\]

\[
CR_{n,n'} \leq C_{n,s} \cdot CR_s, \quad DR_{n,n'} \leq C_{n,s} \cdot \bar{DR}_s
\]

It is worth noting while a heat storage can be charged and discharged simultaneously (through different cycles), an electrical storage cannot. Therefore, it is common to use binary operational variables for an electrical storage unit to prevent simultaneous charging and discharging, e.g. [25]. However, since we consider non-ideal charging and discharging efficiencies (efficiency less than 100%), the model does not need to include such binary variables, as the optimization inherently picks a charging/discharging mode at each step and avoids simultaneous charging/discharging, in order to minimize the cost associated with charging/discharging loss. Our approach results in the same solution (given the optimization is solved with a good accuracy), while using fewer decision variables and constraints. More specifically, the proposed approach saves one binary decision variable and one constraint per node per time-step.

The dispatch of each technology does not exceed its maximum capacity and/or potential, or fall below the minimum acceptable limit (16)-(19).

\[
P_{g,n,c,t} \leq C_{n,c} \cdot SP_t; \quad c \in \{PV, ST\}
\]

\[
n_0 \cdot P_{g,n,t} \leq P_{g,n,t} \leq n_0 \cdot \bar{P}_g
\]

\[
n_0 \cdot n_{n,t} \leq n_{n,t}
\]

\[
P_{g,n,c,t} \leq C_{n,c} \leq b_{n,k} \cdot M
\]

To ensure economic feasibility of the microgrid design, more constraints may be integrated into the model, in which: a) saving in the operational cost of the microgrid is calculated against a base-case (i.e., business-as-usual case); and b) an investment payback constraint is defined that takes into account investment cost, operational cost saving, interest rate, and lifetime of various technologies. Detailed discussion of payback constraints can be found in [25].

**IV. MATHEMATICAL MODEL FOR SECURITY CONSTRAINTS**

In our proposed security-constrained design approach, a series of constraints are integrated into the optimization to ensure the system has enough reserve generation online to make up for the loss of any single generation or storage unit, considering ramping constraints. It is assumed that thermal loads in the system are not critical and the system can tolerate their curtailment. Consequently, only electrical contingencies are taken into account, and thermal generation and storage outages are not considered. Furthermore, following an electrical contingency, the microgrid must supply electrical end-use loads, and thermal end-use loads may be curtailed without a penalty.

As shown in Fig. 3, when an outage happens, the remaining generators (and storage devices) are given some time (Δt\text{curtailed}) to ramp up. After this period, the total system generation must be enough to meet the loads (considering any load curtailment).

We model outage of continuous and discrete technologies differently, as depicted in Fig. 4:

- When considering a continuous technology (e.g. photovoltaic) outage at a node, its entire generation (P_{g,n,c,t}) at the node will be lost. That is because it is assumed that the entire capacity (C_{n,c}) is installed in one unit.
- In contrast, since we allow for several units from a discrete technology (n_{i,n,k}) to be installed at a node, only a portion (generation of a single unit) of the aggregate generation (P_{g,n,c,t}) is lost in an outage. Therefore, although only the aggregate generation of a technology at a node (P_{g,n,c,t}) is relevant for power flow modeling, knowledge about generation of individual units is needed for contingency analysis.
partial load unit is to enable disaggregation of any arbitrary value of $P_{g,n,t}$ into the three categories.

Equations (20)-(22) show how we disaggregate $P_{g,n,t}$ and $n_{o,n,t}$ into the number of units operating at the minimum load ($n_{m,n,t}$), number of units operating at the maximum load ($n_{x,n,t}$), and generator power of the unit operating at a partial load ($P_{g,n,t}^{prt}$). Binary variable $b_{p,n,t}$ denotes whether a partial load unit exists. To simplify the contingency constraints, it is assumed that a partial load unit always exists ($b_{p,n,t} = 1$) if $n_{o,n,t} > 0$ (23).

$$n_{o,n,t} = n_{m,n,t} + n_{x,n,t} + b_{p,n,t}$$

(20)

$$P_{g,n,t} = P_{g,n,t}^{prt} + n_{m,n,t} \cdot P_{g}^{m} + n_{x,n,t} \cdot P_{g}^{x}$$

(21)

$$b_{p,n,t} \cdot P_{g}^{x} \leq P_{g,n,t}^{prt} \leq b_{p,n,t} \cdot P_{g}$$

(22)

$$b_{p,n,t} \cdot M \geq n_{o,n,t}$$

(23)

Next, we develop equations that determine the maximum possible contribution of each DER to the post-contingency state of the system. The maximum output change for minimum load units after a contingency happens, $\Delta P_{g,n,t}^{min}$, is limited by the unused capacity of the units (24) and their maximum rate of change (25). Similarly, the maximum contribution of a partial load unit, $\Delta P_{g,n,t}^{prt}$, is limited by its unused capacity (26) and the maximum rate of change (27). Maximum load units cannot increase their generations if a contingency happens.

The output change for an electric storage system is limited by its maximum ramp rate (28). Also, since the post-contingency dispatch must be sustainable for a period of $\Delta t^{exp}$ (see Fig. 3), storage system must be able to keep its post-contingency output for this period (29). Since renewable generation technologies are assumed to be operating at their maximum potential at all times, these units cannot increase their generation in the post-contingency state.

$$\Delta P_{g,n,t}^{min} \leq (\bar{P}_{g} \cdot P_{g}^{m}) \cdot n_{m,n,t}$$

(24)

$$\Delta P_{g,n,t}^{min} \leq P_{g,n,t}^{prt} \cdot \bar{P}_{g} \cdot \Delta t^{exp} \cdot n_{m,n,t}$$

(25)

$$\Delta P_{g,n,t}^{prt} \leq P_{g,n,t}^{prt} \cdot \bar{P}_{g} - P_{g,n,t}^{prt}$$

(26)

$$\Delta P_{g,n,t}^{prt} \leq P_{g,n,t}^{prt} \cdot \bar{P}_{g} \cdot \Delta t^{exp} \cdot b_{p,n,t}$$

(27)

$$\Delta P_{b,n,s=ES,t} \leq \Delta R_{s=ES} \cdot \Delta t^{exp} \cdot n_{s=ES} - DR_{s=ES,t}$$

(28)

$$\Delta t^{exp} \cdot (\Delta P_{b,n,s=ES,t} + DR_{s=ES,t}) \leq E_{s=ES,t}$$

(29)

To integrate reserve equations into the security-constrained optimal dispatch, the traditional approach [31], [33] is to impose a fixed (static) reserve requirement, such as size of the largest system generator or a percentage of total system load. However, dynamic allocation of system reserve [34], [35] can offer a less conservative and more economical solution. To this end, we develop a novel set of reserve equations that dynamically allocate reserve in the system depending on the generation of different technologies, storage system state of charge, technology ramping constraints, etc. The dynamic reserve equations (security constraints) against outage of renewable (continuous) generation technologies, storage
technologies, and conventional (discrete) generation technologies are presented in (30)-(32), respectively. For each outage, the pre-contingency generation power of the outaged unit must be less than the available reserve in the system, where the available reserve is composed of:

- generation increase in discrete technologies (minimum load units and partial load unit);
- increase in storage output; and
- post-contingency load curtailment including electric chiller load and storage charging load (since they will not be met during the contingency period).

The reserve capacity in the system must be larger than the generation of each continuous unit at each bus n at any given time t (30), where all of the system discrete generators and batteries can contribute to the system reserve. Furthermore, post-contingency electrical load curtailments ($P_{t,n}'$), as well as pre-contingency storage charging and electric chiller loads that will be shed, can be leveraged to increase the system reserve. Equation (31) imposes a similar constraint considering the outage of the storage system at any bus n. Note that all of the system batteries, except for the battery whose outage is being constrained (hence the negative term in the reserve calculation), can contribute to the reserve.

$$P_{g,n,c=PV,t} \leq \sum_{n',g,t} \left( \Delta P g_{n',g,t}^{Min} + \Delta P g_{n',g,t}^{Prt} \right)$$

$$+ \sum_{n'} \Delta P b_{n',s=ES,t}$$

$$+ \sum_{n'} P_{t,n}'$$

$$+ \sum_{n'} CR_{n',s=ES,t} + \sum_{n'} \frac{1}{C_{Pe,c}} \cdot P_{g,n',c=ES,t}$$

$$DR_{n,s=ES,t} \leq \sum_{n',g,t} \left( \Delta P g_{n',g,t}^{Min} + \Delta P g_{n',g,t}^{Prt} \right)$$

$$+ \sum_{n'} \Delta P b_{n',s=ES,t} - \Delta P b_{n,s=ES,t}$$

$$+ \sum_{n'} P_{t,n}'$$

$$+ \sum_{n'} CR_{n',s=ES,t} + \sum_{n'} \frac{1}{C_{Pe,c}} \cdot P_{g,n',c=ES,t}$$

$$P_{g,n,Lat} \leq \sum_{n',g,t} \left( P_{g,n',g,t}^{Min} + \Delta P g_{n',g,t}^{Prt} \right)$$

$$- \Delta P g_{n,Lat}$$

$$+ \sum_{n'} \Delta P b_{n',s=ES,t}$$

$$+ \sum_{n'} P_{t,n}'$$

$$+ \sum_{n'} CR_{n',s=ES,t} + \sum_{n'} \frac{1}{C_{Pe,c}} \cdot P_{g,n',c=ES,t}$$

In (32) for imposing the security constraint for discrete generator outages, $P_{g,n,Lat}$ is the largest generation power among all of the units from technology g connected to bus n. Hence, $P_{g,n,Lat}$ is $P_{g}$ if there is a unit operating at maximum load (33)-(34), or otherwise, is the power of the partial load unit (35). The variable $\Delta P g_{n,Lat}$ is used to negate the contribution of the outaged unit from the total contribution of the units. It is 0 if the outaged unit is a maximum load unit (when $bx_{n,g,t} = 0$), as shown in (36).

$$P_{g,n,Lat} \geq bx_{n,g,t} \cdot P_{g}$$

$$bx_{n,g,t} \leq n_g \cdot \delta$$

$$P_{g,n,Lat} \geq P_{g,n,Prt}$$

$$\Delta P g_{n,Lat} \geq \Delta P g_{n,Prt} - bx_{n,g,t} \cdot P_{g}$$

V. CASE STUDY

A. Case setup

We developed an isolated microgrid test system based on a real-life remote utility microgrid in Nome, Alaska [36]. This model is composed of 19 nodes and its GIS and electrical and thermal single line diagrams are shown in Fig. 5. The important technology parameters are shown in Table I and Table II. More details can be provided upon request.

We consider three cases. In Case I the objective is to minimize overall costs, but N-1 contingency constraints are not included. In Case II, we minimize the overall costs while including the contingency constraints. In Case III, a composite objective (50% weight for cost objective and 50% weight for CO2 emission objective) is considered and security constraints are also taken into account. In each case, the optimization model determines the optimal mix and size of discrete (1,000 and 5,000 kVA CHP-enabled diesel engines) and continuous (photovoltaic, battery, boiler, and electric chiller) technologies that can be installed at nodes 1, 8, and 18, which are the microgrid’s central power plant, a hospital, and a residential neighborhood toward the end of a long feeder, respectively.
In order to -d cannot contribute to the -olution includes -em security against generator outage. First, i

19% increase in the total annual investment and operation costs. in a 14% reduction in CO₂ emissions. The reason is that the larger unit has a higher efficiency (41.6% vs. 36.8%) and hence, can reduce CO₂ emissions. The optimal DER mix in Case III results in a 14% reduction in CO₂ emissions that is made possible by a 19% increase in the total annual investment and operation costs.

### TABLE I
INVESTMENT PARAMETERS FOR CONTINUOUS TECHNOLOGIES

<table>
<thead>
<tr>
<th>Technology</th>
<th>Fixed Cost ($)</th>
<th>Variable Cost ($/kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery</td>
<td>75,000</td>
<td>500</td>
</tr>
<tr>
<td>Photovoltaic</td>
<td>40,000</td>
<td>4,000</td>
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<tr>
<td>Boiler</td>
<td>4,500</td>
<td>30</td>
</tr>
<tr>
<td>Electric Chiller</td>
<td>2,300</td>
<td>230</td>
</tr>
</tbody>
</table>

### TABLE II
INVESTMENT AND OPERATION PARAMETERS FOR DISCRETE TECHNOLOGIES

<table>
<thead>
<tr>
<th>Technology</th>
<th>Size (kW)</th>
<th>Cost ($/kW)</th>
<th>Eff. (%)</th>
<th>Min/Max Load (%)</th>
<th>Ramp Rt. (%/min)</th>
<th>Ht. Rec. Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHP Diesel</td>
<td>1,000</td>
<td>1,911</td>
<td>36.8%</td>
<td>30%, 100%</td>
<td>50%</td>
<td>1.019</td>
</tr>
<tr>
<td>CHP Diesel</td>
<td>5,000</td>
<td>1,182</td>
<td>41.6%</td>
<td>30%, 100%</td>
<td>50%</td>
<td>0.797</td>
</tr>
</tbody>
</table>

### B. Optimal DER mix, capacity, and siting

The results of the case studies, including DER capacities at each node (i.e., n1, n8, and n18), annualized investment and annual operation costs, post-contingency load curtailment costs, and annual CO₂ emissions are summarized in Table III.

In Case I, only one unit of the 5,000 kVA CHP-enabled diesel generator is installed at the central power plant (node 1) without any photovoltaic or battery investments in the system. When the contingency constraints are added to the optimization model in Case II, a 2,833 kWh battery storage system is added to the optimal technology mix. Furthermore, the solution includes several small diesel units instead of one large unit (3×1,000 vs. 1×5,000 kVA), because several units operating in parallel enhance the system security against generator outage contingencies. Consequently, both investment and operation costs increase in Case II. It is worth noting that the optimal solution does not include any photovoltaic systems, mainly because they are not dispatchable and cannot contribute to the reserve constraints.

In Case III, cost minimization and emission reduction are considered with the same weight. Several observations can be made from the optimal investment solution. First, in order to reduce CO₂ emissions, 4,274 kW of photovoltaics are installed at node 8, although photovoltaic was not cost-effective in Case I and II. Second, the size of the battery storage system at node 8 becomes threefold larger compared to Case II. Third, compared to Case II, one of the 1,000 kVA diesel units at node 1 is replaced with a 5,000 kVA unit. The reason is that the larger unit has a higher efficiency (41.6% vs. 36.8%) and hence, can reduce CO₂ emissions. The optimal DER mix in Case III results in a 14% reduction in CO₂ emissions that is made possible by a 19% increase in the total annual investment and operation costs.

### TABLE III
CASE STUDY RESULTS – OPTIMAL TECHNOLOGY MIX AND SITING

<table>
<thead>
<tr>
<th>DER Capacities</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photovoltaic</td>
<td>n1</td>
<td>n8</td>
<td>4,274</td>
</tr>
<tr>
<td></td>
<td>n18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td>n1</td>
<td>n8</td>
<td>2,883</td>
</tr>
<tr>
<td></td>
<td>n18</td>
<td></td>
<td>8,220</td>
</tr>
<tr>
<td>Diesel Engine</td>
<td>n1</td>
<td>1×5,000</td>
<td>3×1,000</td>
</tr>
<tr>
<td></td>
<td>n8</td>
<td>2×1,000</td>
<td>2×1,000 + 1×5,000</td>
</tr>
<tr>
<td></td>
<td>n18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Costs</td>
<td>Operation</td>
<td>8,133</td>
<td>8,662</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>962</td>
<td>1,240</td>
</tr>
<tr>
<td></td>
<td>Curtailment</td>
<td>-</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>9,096</td>
<td>9,932</td>
</tr>
<tr>
<td>CO₂ (tons)</td>
<td>16,104</td>
<td>16,650</td>
<td>14,329</td>
</tr>
</tbody>
</table>

### C. Optimal electricity, heating, and cooling dispatch

To illustrate dispatch signals, Fig. 6 - Fig. 9 show optimal electrical and heating dispatch for multiple nodes in Case III. The optimal electrical dispatch for nodes 1 and 8 is shown in Fig. 6 and Fig. 7, respectively. Export of energy (to other nodes) is shown with negative values. The generation from the 3×1,000 kVA and 1×5,000 kVA diesel generators at node 1 is entirely exported to other nodes, since the node does not have any loads of its own. Node 8 is equipped with a photovoltaic and a battery system to supply its own load and export the extra power to other nodes. The battery is charged during peak photovoltaic hours and discharged at morning and afternoon. The interplay between electricity and cooling impacts the load in this figure, since it includes the electrical end-use loads as well as electrical consumption of chillers.

![Fig. 6. Optimal electricity dispatch for node 1 (Case III, September weekday)](image)
The optimal heating dispatch for nodes 1 and 11 is shown in Fig. 8 and Fig. 9, respectively. The heat from diesel units at node 1 is recovered and exported to other nodes through pipes. The consideration of recovered heat ties the dispatch of electricity and heating in the system. The heating loads at node 11 are met by the heat imported to the node. This node also exports the extra heat (from import) to other microgrid nodes.

D. Security against N-1 contingencies

Fig. 10 shows the generation outage power vs. system reserve for various generation contingencies during a September peak day in Case III (arbitrarily chosen), where the outage power and system reserve refer to the left-hand side and right-hand side terms in (30)-(32), respectively. As shown in this figure, the integration of security constraints forces the system reserve to be more than the outage power at any given time. In this example, outage of the 5,000 kVA diesel unit at node 1 is the most severe contingency, since it is much larger than all other dispatchable units in the system. Fig. 10(d) confirms this intuition and shows that although the system maintains enough reserve against this contingency, the difference between the outage power and the reserve in the system is much smaller compared to other contingencies.

E. Voltage profile and linear power flow accuracy

The box plot for voltage magnitude error at each microgrid node is shown in Fig. 11 for Case III, where the error is the % difference between the approximate voltage solution (from the optimization) and the exact voltage solution (obtained post-optimization using Newton-Raphson algorithm). The error increases as the node distance from the slack bus, i.e. node 1, increases. However, the maximum error is less than 0.6%, which shows a very high accuracy. Our analysis shows that in this case, >87% of voltage data points have an error less than 0.3% and >97% of data points have an error less than 0.5%.
VI. CONCLUSIONS AND FUTURE WORK

This paper presents a novel approach for security-constrained optimal design of isolated multi-energy microgrids, formulated as a mixed integer linear program. Our optimal design entails optimal DER technology mix, size, and dispatch, and takes into account the interplay between electricity, heating, and cooling loads and sources in the system; and hence, is able to capture benefits of CHP technologies. To provide a secure design/operation against N-1 generator contingencies, a novel model was developed to dynamically assess the microgrid generation reserve.

To illustrate how the method works, several case studies were carried out on an isolated microgrid model that we developed based on a real isolated utility microgrid in Alaska and the impact of contingency constraints on the optimal solution was discussed. The studies showed that a cost-minimization design, especially in the presence of security constraints, may not lead to adoption of renewable resources, mainly due to their un-dispatchability and inability to ramp up generation following a system contingency. However, a cost-emission composite objective may lead to deployment of renewable technologies. Evaluating the accuracy of the integrated linear power flow equations (LinDistFlow) showed very high accuracy for the model.

Future research will focus on integrating network design (cable/pipe placement and sizing) into the model. We will also explore how renewable generation stochasticity can be incorporated into the optimization model. Furthermore, integration of non-electrical contingencies, e.g. outage of thermal resources, can also add to the value of this work.

VII. ACKNOWLEDGMENT

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VIII. REFERENCES


