DA-248
LBL-25124

Presented at the 13th National Passive Solar Conference, June 19-24, 1988, Massachusetts Institute of Technology, and published in the proceedings.

Determination and Application of Bidirectional Solar-Optical Properties of Fenestration Systems

K. Papamichael, J. Klems, and S. Selkowitz

Windows and Daylighting Group
Applied Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

March 1988

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Buildings and Community Systems, Buildings Systems Division of the U.S. Department of Energy under Contract No. DE-ACO3-76SF00098, with cosponsorship by the Electric Power Research Institute and the New York State Energy Research and Development Authority under Contract No. RP2418-5-1, under the management of the Lighting Research Institute, Contract No. 85:IMP:1.

Determination and Application of Bidirectional Solar-Optical Properties of Fenestration Systems

K. Papamichael J. Klems S. Selkowitz

Windows and Lighting Program
Center for Building Science
Applied Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

Abstract

Accurate determination of the luminous and thermal performance of fenestration systems that incorporate optically complex components requires detailed knowledge of their radiant behavior. We describe a large scanning radiometer used to measure the bidirectional transmittance and reflectance of fenestration systems and components. We present examples of measured data obtained for simple non-specular samples. We describe a method of obtaining the overall properties of fenestration systems by calculation from scanning radiometer measurements of fenestration components. Finally, we describe the application of bidirectional solar-optical properties of fenestration systems to determine their luminous and thermal performance with respect to building energy consumption and occupants' comfort. We also discuss the advantages and limitations of the method, which appears to be promising.

1. Introduction

Fenestration systems affect building energy use in two ways. First, daylight admitted through the fenestration may reduce the need for electric lighting and hence reduce electricity usage. Second, the solar energy admitted may either reduce heating loads or increase air conditioning loads, depending on the season of the year and on other characteristics of the building such as internal loads. To calculate the reduction of electric lighting requirements, we need a detailed knowledge of the spatial distribution of daylight within the interior space to determine the illumination levels available for the particular visual tasks to be performed. This requires knowledge of the angular distribution of the light admitted through the fenestration for any exterior sun, sky and ground conditions. While in principle determining the effect of solar heat gain also requires knowledge of the distribution of the energy within the space, since absorptivities and thermal capacities of surfaces may vary, in practice the available building energy simulation models utilize simplified calculations that require only the total amount of admitted solar energy.

For simple fenestration systems consisting of one or more layers of clear, tinted or coated glass, where the principal optical effects are absorption, unidirectional transmission and specular reflection, standard calculation procedures (1, 2) based on photometric measurements of material optical properties (3) are adequate to predict the transmission of the system. For these systems computer programs for calculating interior daylight levels also exist (4). However, in many cases layers such as blinds, louvres or drapes that are spatially inhomogeneous and/or diffusely transmitting or reflecting are used to control solar heat gain and daylighting levels. For these systems the standard procedures are not adequate.

At Lawrence Berkeley Laboratory we are establishing a method to accurately predict the luminous and thermal performance of fenestration systems that incorporate optically complex components. This method is based on combining experimental procedures to determine detailed, angle-dependent, solar-optical properties of fenestration components and computational routines to determine the luminous and thermal performance of fenestration systems using the detailed solar-optical properties of their components. In this paper we describe an apparatus for measuring the bidirectional solar-optical properties of fenestration components and systems, and present results from measurements of the bidirectional transmittance and reflectance of simple fenestration components. We present and discuss the determination of the solar-optical properties of fenestration systems from the properties of their components. Finally, we describe the use of the solar-optical properties of fenestration systems to determine their luminous and thermal performance with respect to building energy consumption and occupants' comfort.

Since we have currently dealt only with the optical properties of fenestration components, we refer only to photometric quantities and symbols. However, our procedures also apply to the total solar spectrum.

2. Bidirectional Solar-Optical Properties of Fenestration Systems

The bidirectional transmittance, $\tau(\theta_o,\phi_o;\theta_i,\phi_i)$, (or reflectance, $\rho(\theta_o,\phi_o;\theta_i,\phi_i)$) of a fenestration component or system is defined as the ratio of the transmitted (or reflected) flux collected over an element of solid angle surrounding the outgoing direction specified by the angles θ_o and ϕ_o to essentially collimated incident flux incoming from the direction specified by the angles θ_i and ϕ_i (Figure 1):

$$\tau(\theta_o,\phi_o;\theta_i,\phi_i) = \frac{dL_o(\theta_o,\phi_o)}{dE_i(\theta_i,\phi_i)} \quad [sr^{-1}], \tag{1}$$

where $dL_o(\theta_o,\phi_o)$ is the element of the outgoing (transmitted or reflected) luminance and $dE_i(\theta_i,\phi_i)$ is the element of the incident illuminance, normal to the incoming direction.[1]

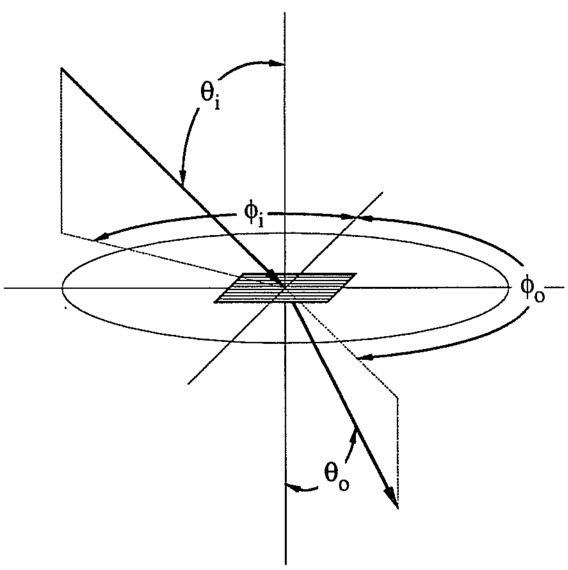

Fig. 1. Definition of angles for bidirectional transmittance.

The luminance of homogeneous layers due to transmitted or reflected radiation is a function of the outgoing direction alone (assuming uniform collimated incident flux). The luminance of inhomogeneous layers, however, is a function of both the outgoing direction and the position on the layer. In this case, we consider an average outgoing luminance over the entire fenestration layer (Figure 2).

[1] We use E_i as the illuminance in front of the sample normal to the incoming direction instead of incident on the sample, to cover devices that transmit or reflect radiation incoming at 90° incident angle, such as overhangs and awnings.

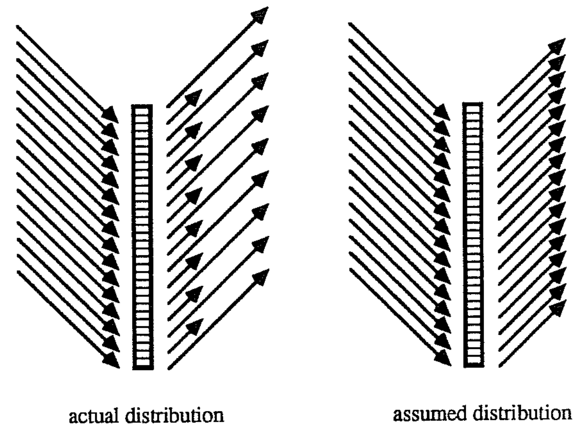

Fig. 2. Consideration of an average luminance over the area of a sample.

There are six solar-optical properties that fully characterize a fenestration layer: the bidirectional front and back transmittance and reflectance (τ^f, τ^b, ρ^f, ρ^b) and the directional front and back absorptance (α^f, α^b). For fenestration systems, however, information about the absorptance of each layer is also essential for determining the contribution of the absorbed radiation to the total solar heat gain through convection/conduction.

3. Determination of Bidirectional Solar-Optical Properties

In order to determine the bidirectional solar-optical properties of fenestration components we have built a large scanning radiometer (Figures 3 and 4). This radiometer, which was originally designed to measure candle-power distributions (5), consists of a fixed-position light source, a sample-holding plane with two rotational degrees of freedom, and a movable detector. The sample plane may be rotated about a vertical axis to adjust the angle of incidence, and about a horizontal axis to adjust the azimuth angle relative to a preferred direction on the device. The full incident hemisphere may be covered in this manner, although in most cases device symmetries will make measurements over the entire hemisphere unnecessary. The detector moves along a vertical semicircular track to cover an arc of 180°, and this track may be rotated about a vertical axis through a full revolution, enabling the detector to move over both the front and back hemispheres of the sample.

The movements of the scanning radiometer are driven by stepper motors under computer control. The detector is driven through its semicircular arc and 120 approximately equally spaced data points are recorded. The detector arm is then rotated horizontally by a pre-set angle and the detector again sweeps through its arc. In this manner the entire outgoing hemisphere in a transmittance or reflectance measurement is scanned, and the computer steps the sample through a grid of incident angles and sample azimuths, scanning the outgoing hemisphere for each. Currently all of the rotation steps, with the exception of the semicircular vertical movement of the detector, are set at 15°; with this step size it takes about 20 minutes to scan a complete hemisphere, that is, to consider a single incoming direction.

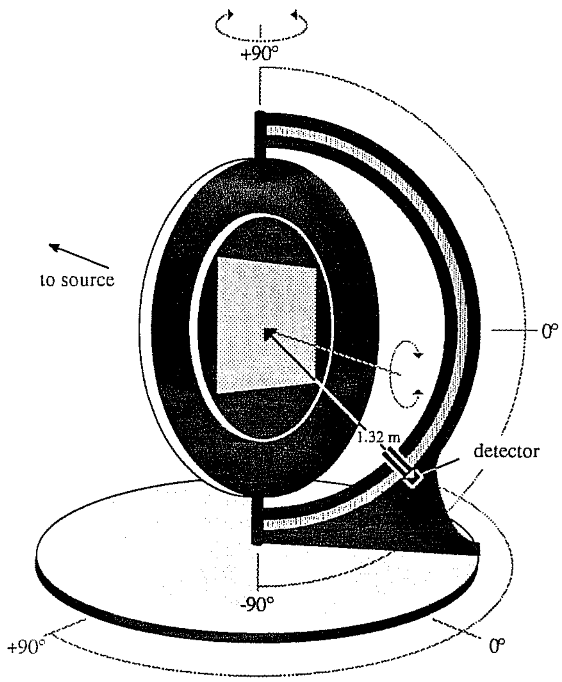

Fig. 3. Schematic of the scanning radiometer, showing the possible rotations of the sample and the convention for the coordinates of the detector.

The movable detector of the radiator is a photopically corrected silicon sensor read by a computer-controlled digital voltmeter. Two additional fixed sensors are used, one to monitor the source intensity and one to record the illuminance in front of the sample, normal to the incoming direction. Data are read into a DEC LSI-11 computer and stored temporarily on a hard disc. At the completion of a measurement run data are transmitted to a VAX computer for analysis. Separate measurements of luminous noise are subtracted and the data points are interpolated to give values on a fixed grid in (θ_o,ϕ_o) coordinates. The average outgoing luminance is then calculated using the geometrical characteristics of the scanning radiometer, assuming that the area of the detector is very small with respect to the detector's distance from the sample and the area of the sample, as

$$\overline{L_o}(\theta_o,\phi_o) = \frac{E_S}{\int_A \frac{dA\cdot\cos\theta_A\cdot\cos\theta_S}{R^2}} \quad [\mathrm{lumens\cdot m^{-2}\cdot sr^{-1}}], \quad (2)$$

where E_S is the illuminance recorded by the moving detector, A is the area of the sample, θ_A and θ_S are the angles between the propagation direction and the normal to the sample and the detector, respectively, and R is the distance from the detector to the sample elements. For small samples, where the angles angles, θ_A and θ_S, and the distance, R, do not change considerably over the area of the sample, equation 2 is simplified to:

$$\overline{L_o}(\theta_o,\phi_o) = \frac{E_S\cdot R^2}{A\cdot\cos\theta_o} \quad [\mathrm{lumens\cdot m^{-2}\cdot sr^{-1}}], \quad (3)$$

where θ_o is the angle between the propagation direction and the normal to the sample at its center.

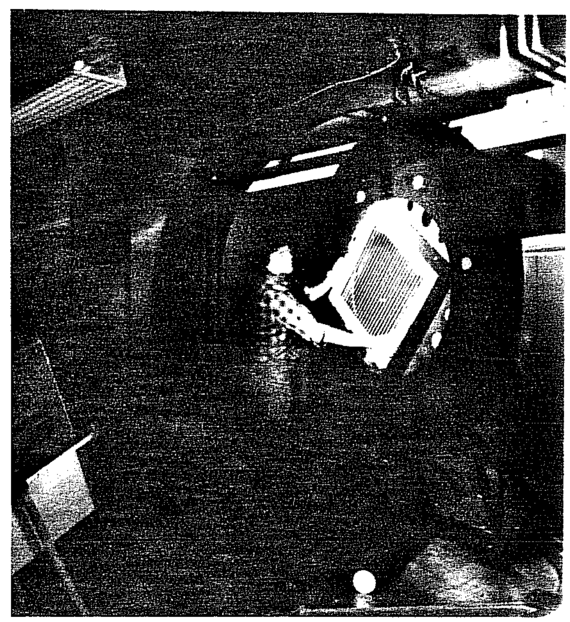

Fig. 4. The Scanning Radiometer during early testing.

Since the detector accepts radiation from the entire area of the sample, the determined outgoing luminance is an average over the area of the sample, with respect to both position on the sample and direction within the solid angle subtended by the sample at the detector. We term this the "equivalent average luminance" of the sample.

Examples of results obtained with the scanning radiometer are shown in figures 5, 6 and 7. Figure 5 shows the transmitted distribution through a diffusive sample for an incident angle of 0°. The data are shown in terms of the detector coordinate angles, prior to their transformation into (θ_o,ϕ_o) coordinates. This graph displays the entire 120-point scan in the sensor altitude, hence the narrow line spacing in that dimension. As expected, the data show a broad peak centered about the incident direction. Figure 6 shows the reflected distribution from the same sample for an incident angle of 45°. The data in this plot have been interpolated in the neighborhood of -30° sensor azimuth, where the sensor arm shadows the sample. The data here indicate a combination of specular and diffuse reflection. Figure 7 shows the transmitted distribution through a white, slatted, venetian-blind-like sample with the slats fully open and an incident angle of 60° (which corresponds to a sensor azimuth of -60° and a sensor altitude of 0°). The incident plane is perpendicular to the slat direction. Here we can clearly see the outgoing distribution resulting from one reflection off the slats (high peak at 30° sensor azimuth) and from two reflections (smaller peak at -45° sensor azimuth).

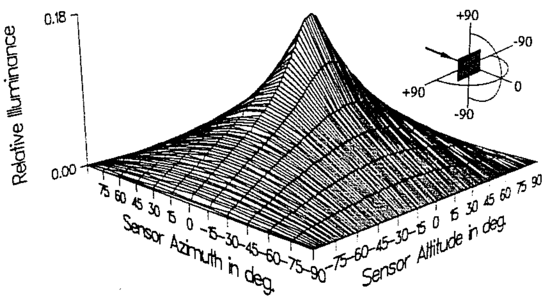

Fig. 5. Transmitted distribution through a diffusive sample for 0° incident angle.

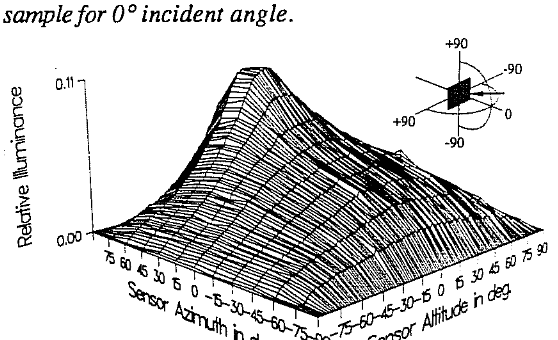

Fig. 6. Reflected distribution from a diffusive sample for 45° incident angle.

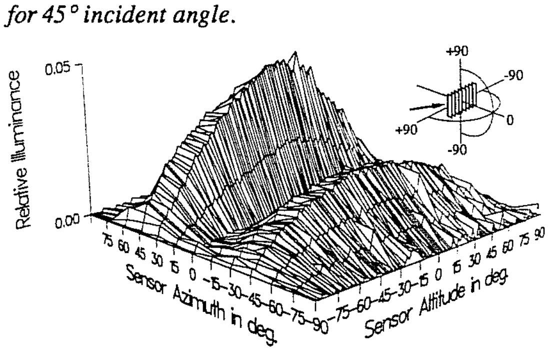

Fig. 7. Transmitted distribution through a white slat-type sample for 60° incident angle on a plane perpendicular to the slat direction.

4. Application of Bidirectional Solar-Optical Properties

Once the bidirectional solar-optical properties of fenestration layers are determined, they are organized into matrices, where the rows and columns correspond to the outgoing and incoming directions, respectively. The solar-optical properties of any combination of layers are then calculated using matrix operations (6).

If we consider a pair of adjacent layers, i and j, and for the moment neglect interreflections between them, then for a given illuminance E_i on layer i, normal to the incoming direction, the total illuminance incident at a particular point on layer j is (Figure 8)

$$E_j = E_i \cdot \int_{\Omega_i} \tau_i^f(\theta_o,\phi_o;\theta_i,\phi_i)\cdot d\Omega_{ij} \quad [\text{lumens}\cdot\text{m}^{-2}], \qquad (4)$$

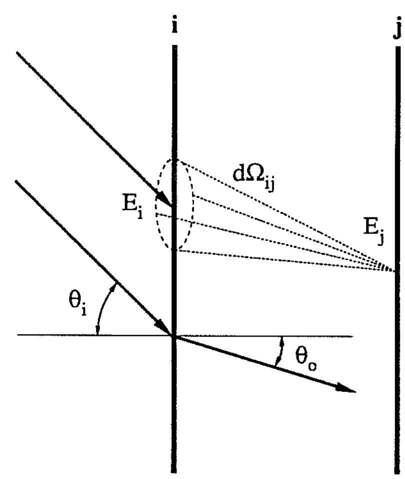

Fig. 8. The propagation of radiation between two adjacent layers of a fenestration system.

where τ_i^f is the front bidirectional transmittance of layer i, dW_{ij} is the solid angle subtended at the point on j by an element of area on i, and the integral is taken over the layer i. If the integral in equation 4 is approximated by a sum over finite elements of solid angle, then the equation may be rewritten in matrix form as

$$E_j = \Omega_i \cdot \tau_i^f \cdot E_i \quad [\text{lumens}\cdot\text{m}^{-2}], \qquad (5)$$

where W_i is a diagonal matrix of the solid angle elements and represents the propagation from layer i to the point on layer j. If multiple reflections between the layers are now included equation 5 becomes

$$E_j = (1 - \Omega_i \cdot \rho_i^b \cdot \Omega_j \cdot \rho_j^f)^{-1} \cdot \Omega_i \cdot \tau_i^f \cdot E_i \quad [\text{lumens}\cdot\text{m}^{-2}], \qquad (6)$$

where the ρ_i^b and ρ_j^f are the respective front and back bidirectional reflectances of the layers. For a two-layer fenestration system the overall front transmittance would then be given by

$$\tau^f = \tau_2^f \cdot (1 - \Omega_1 \cdot \rho_1^b \cdot \Omega_2 \cdot \rho_2^f)^{-1} \cdot \Omega_1 \cdot \tau_1^f \qquad [sr^{-1}], \qquad (7)$$

or

$$\tau^f = \tau_2^f \cdot R^f \cdot \Omega_1 \cdot \tau_1^f \qquad [sr^{-1}], \qquad (8)$$

where R^f represents the interreflections between the two layers (Figure 9).

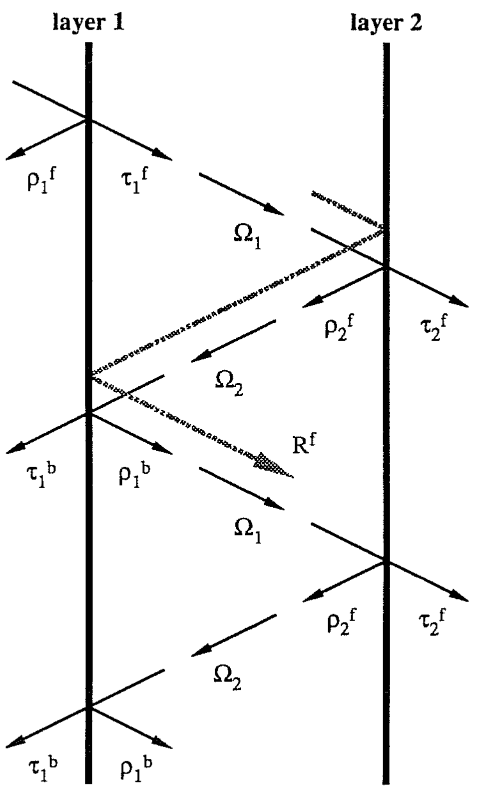

Fig. 9. Multiple reflections between two fenestration layers.

Systems consisting of more than two layers are more complicated in that reflections from subsequent layers incident on the back of layer j must be included, but these do not change the essential point that the system properties may be calculated from the individual layer bidirectional properties through a series of matrix operations.

A computer program, named "TRA", has been developed to perform the above-described matrix operations for a two-layer fenestration system. It is used to compute the bidirectional solar-optical properties of the fenestration system, as well as the directional-hemispherical properties of the two layers and of the system, including the directional absorptance of each layer as part of the fenestration system.

The bidirectional optical properties of fenestration systems are used by the daylight analysis computer program SUPERLITE (7, 8) in a series of parametric analyses. The parameters varied refer to the sun position, the fenestration size and location, the reflectance of the interior surfaces, and the location on the work plane. The results from these parametric analyses are then statistically analyzed to produce daylight coefficients-of-utilization for radiation that is direct, diffuse from the sky and diffuse from the ground. These coefficients-of-utilization are regression equations that can be used to quickly estimate work plane illuminance from the values of the relevant parameters mentioned above (9).

The directional-hemispherical solar properties are used by a computer program named SSG (Sun Sky Ground) in integration over a large number of sky luminance distributions to determine average transmittance and absorptance values for radiation that is direct, diffuse from the sky and diffuse from the ground. These average values, along with experimentally determined heat transfer correlations, will then be used by a new version of the computer program WINDOW (10), to calculate solar heat gain coefficients and U-values for a large number of outdoor and indoor environmental conditions. The output of WINDOW will then be statistically analyzed to express solar heat gain coefficients and U-values as functions of the relevant environmental parameters.

The daylight coefficients-of-utilization along with the solar heat gain and U-value functions will then be supplied to the energy analysis computer program DOE-2.1C (11, 12), to calculate energy use patterns for every hour of the year. The experimental and computational process (Figure 10) includes validation stages using our integrating sphere (13), our sky simulator (14) and our Mobile Window Thermal Test (MoWiTT) facility (15).

5. Discussion

This approach represents a new step towards the simulation of the luminous and thermal performance of fenestration systems that incorporate optically complex components. Although this approach requires significant effort and computational power, we believe that it contributes greatly to the proper assessment of the luminous and thermal performance of complex fenestration systems, which cannot be achieved otherwise. The computation of the properties of fenestration systems from the properties of their components contributes towards reducing the effort and information requirements of lengthy measurement procedures for the large number of fenestration systems

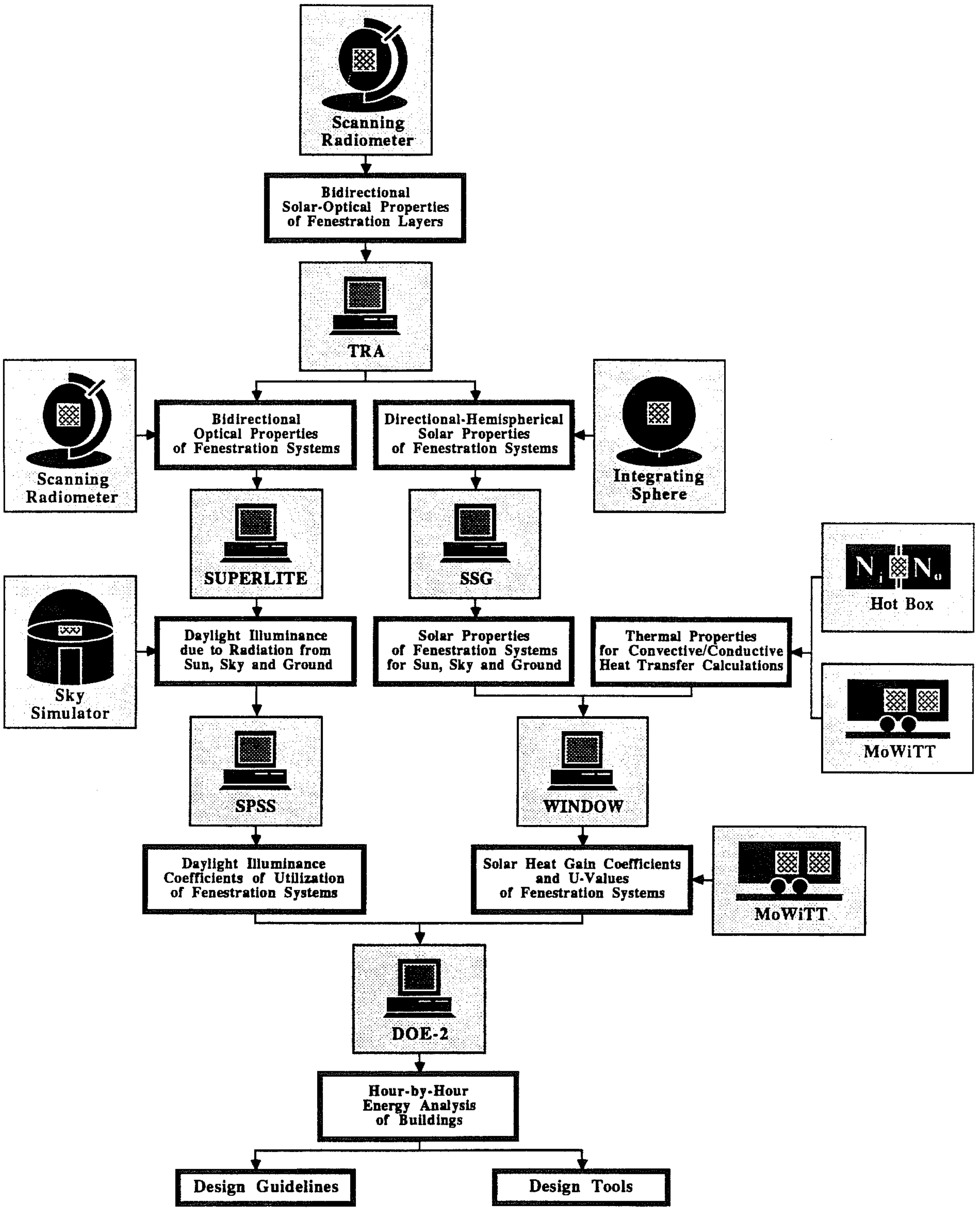

Fig. 10. Overall scheme for producing and using bidirectional solar-optical properties of fenestration systems, towards the development of simplified design guidelines and tools.

which may result from combinations and permutations of even a small number of components. Moreover, this approach offers essential information on the absorbed radiation by layer which is otherwise unobtainable. However, the validity of the underlying assumptions and the utility of the approach are as yet untried, and a thorough validation of the procedure is necessary.

A key assumption is the concept of equivalent average luminance, expressed in the statement that the spatial variations in optical properties may be averaged over the device dimensions without changing the resulting lighting or heat gain calculations. This is a reasonable assumption for fenestration systems, since absorption of solar energy at surfaces is itself a spatial averaging process, and since good lighting design will allow spatially irregular light fluxes to reach a visually important surface only after at least one diffuse reflection, which similarly averages out the spatial variation. However, to extend this assumption to the individual layers is a much stronger statement. While this assumption is not valid for some systems, one purpose of our work is to determine whether it holds for a usefully large class of fenestration systems.

A second assumption is that the angular resolution of the scanning radiometer is sufficient for a usefully large class of shading devices. The angular resolution is limited by the sample size, the detector size, and the sample-detector distance. For our apparatus, the latter was dictated by cost considerations. It is very difficult to determine a priori the angular resolution necessary. Only tests of the method for a variety of realistic components and systems will answer this question.

A third assumption is that the computational power necessary to carry out the matrix calculations remains reasonable. This hinges strongly on the degree of angular accuracy necessary. With the current 15° angular grids the bidirectional matrices contain 145 x 145 elements, and as is well-known, computation time increases very rapidly with matrix size. For systems of more than two layers, multiple reflections between non-adjacent layers will cause the calculation time to rise faster than linearly with the number of layers. Also, simple storage, indexing and accessing of the measured properties becomes a problem with the volume of information used by this method. However, a simplifying circumstance is that most of the layers in any fenestration system will be glazings, for which the matrices are diagonal. It is likely that even the most complex fenestrations will not have more than three or four optically complex layers, and many will have only one.

In carrying out the proposed approach to characterizing fenestration systems we will investigate all of the above issues. Moreover, several validation procedures will be followed (Figure 10). Two of these are directly related to determining the solar-optical properties of fenestration systems. The first validation procedure is based on comparing the directional-hemispherical transmittance obtained by integration of measured bidirectional transmittance over the output hemisphere, with that measured directly using our large integrating sphere (11). This comparison has so far been carried out only for the transmittance of a uniform diffusing sample. For an incident angle of 45° the directional-hemispherical transmittance was calculated to be 0.51 and a somewhat crude measurement with the integrating sphere yielded a value of 0.47. The difference is within the estimated experimental error of the sphere measurement. The second validation procedure (not shown in Figure 10) is based on comparing measured bidirectional properties of fenestration systems, using the scanning radiometer, with those calculated using the TRA computer program from measured layer properties.

6. Conclusions

We have succeeded in constructing and operating a large scanning radiometer capable of rapidly and economically producing optical data, otherwise unobtainable, for optically complex fenestration components. While the data presented here are preliminary and calibration and extension to the radiometric regime are still to be completed, the facility already represents a unique measurement capability.

The method of characterizing layers by their equivalent average luminance and combining separately measured properties by calculation has the potential for solving an otherwise difficult combinatorial problem in characterizing fenestration systems. While the range of applicability of this method is still to be determined, it is clearly useful for the large class of fenestration systems consisting of a single geometrically complex shading device in combination with several glazing layers, with or without tints or coatings. We believe that it has considerable promise.

7. Acknowledgements

The authors are indebted to Ross McCluney for valuable discussions on the theoretical basis of the method, to Mark Spitzglas for his work on the initial design and construction of the scanning radiometer, and to Dennis DiBartolomeo, Mary Hinman and Mehrangiz Yazdanian for their work on the automated control and data acquisition capability of the scanning radiometer.

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Buildings and Community Systems, Buildings Systems Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, with cosponsorship by the Electric Power Research Institute and the New York State Energy Research and Development Authority under Contract No. RP2418-5-1, under the management of the Lighting Research Institute, Contract No. 85:IMP:1.

8. References

(1) ASHRAE, *ASHRAE Handbook 1985 Fundamentals*, Chapter 27, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, Georgia (1985).

(2) Rubin, M., "Calculating Heat Transfer Through Windows," *Energy Research*, Vol. 6, pp. 341-349 (1982).

(3) Rubin, M., "Optical Properties of Soda Lime Silica Glasses," *Solar Energy Materials*, 12, North-Holland, Amsterdam, pp. 275-288 (1985).

(4) Windows and Daylighting Group, "Daylighting Design Tool Survey," Lawrence Berkeley Laboratory (1986).

(5) Spitzglas, M., "Defining Daylighting from Windows in Terms of Candlepower Distribution Curves," Lawrence Berkeley Laboratory Report LBL-18087 (1984).

(6) Papamichael, K., Winkelmann, F., "Solar-Optical Properties of Multilayer Fenestration Systems," 1986 International Daylighting Conference Proceedings I, Long Beach, CA (Lawrence Berkeley Laboratory Report LBL-20543) (1986).

(7) Modest, M., "A General Model for the Calculation of Daylighting in Interior Spaces," *Energy and Buildings*, Vol. 5, pp. 69-79 (1982).

(8) Kim, J.J., Papamichael, K., Spitzglas, M., Selkowitz, S., "Determining Daylight Illuminance in Rooms Having Complex Fenestration Systems," 1986 International Daylighting Conference Proceedings I, Long Beach, CA (Lawrence Berkeley Laboratory Report LBL-20542) (1986).

(9) Kim, J.J., Papamichael, K., Selkowitz, S., "Development of Regression Equations for A Daylight Coefficient-of-Utilization Model," 1986 International Daylighting Conference Proceedings I, Long Beach, CA (Lawrence Berkeley Laboratory Report LBL-20539) (1986).

(10) Windows and Daylighting Group, "WINDOW 2.0 User and Reference Guide," Lawrence Berkeley Laboratory (1987).

(11) Building Energy Simulation Group, "DOE-2 Supplement: Version 2.1C," Lawrence Berkeley Laboratory, Report LBL-8706, Rev. 4 Suppl. (1984).

(12) Building Energy Simulation Group, "Overview of the DOE-2 Building Energy Analysis Program," Lawrence Berkeley Laboratory, Report LBL-19735, UC-95d (1985).

(13) Kessel, J., Selkowitz S., "Integrating Sphere Measurements of Directional-Hemispherical Transmittance of Window Systems," *Journal of the Illuminating Engineering Society*, 14, No 1 (Lawrence Berkeley Laboratory Report LBL-17284) (1984).

(14) Selkowitz, S., "A Hemispherical Sky Simulator for Daylighting Model Studies," Proceedings of the 6th National Passive Solar Conference, eds., J. Hayes, W. Kolar, pp. 850-854, American Section, International Solar Energy Society, Inc. (Lawrence Berkeley Laboratory Report LBL-12967) (1981).

(15) Klems, J., Selkowitz, S., Horowitz, S., "A Mobile Facility for Measuring Net Energy Performance of Windows and Skylights," *Energy Conservation in the Built Environment*, Vol. III, Proceedings of the 3rd International Symposium on CIB Working Commission W-67, Dublin, Ireland: An Foras Forbatha (1982).

TA-233
LBL-20543

Presented at the 1986 International Daylighting Conference, 5-7 November 1986, Long Beach, CA, and published in 1986 International Daylighting Conference Proceedings II (technical proceedings), 1988.

Solar-Optical Properties of Multilayer Fenestration Systems

Konstantinos Papamichael and Fred Winkelmann

Center for Building Science
Applied Science Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, CA 94720

November 1986

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Buildings and Community Systems, Building Systems Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and the Electric Power Research Institute and the New York State Energy Research and Development Authority under Contract No. RP2418-5-1, under the management of the Lighting Research Institute, Contract No. 85:IMP:1.

TA-233
LBL-20543

Solar-Optical Properties of Multilayer Fenestration Systems

Konstantinos Papamichael and Fred Winkelmann
Center for Building Science
Applied Science Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, CA 94720

ABSTRACT

The bidirectional solar-optical properties of a fenestration system are necessary to accurately determine its luminous and thermal performance. Bidirectional transmittance and reflectance can be determined experimentally for fenestration systems of arbitrary complexity using a scanning radiometer, after which the total directional absorptance can be calculated. However, for the case of multilayer fenestration systems, this approach does not provide information about the net absorptance of each layer. Moreover, the same layers can be ordered in more than one way, resulting in fenestration systems with different solar-optical properties, the determination of which requires additional experimental procedures. This paper describes a mathematical model for the calculation of the bidirectional solar-optical properties of multi-layer fenestration systems, using the bidirectional solar-optical properties of each layer. The model is based on the representation of the bidirectional solar-optical properties using matrices. Matrix operations are then used to calculate the bidirectional solar-optical properties of any combination of layers, considering the interreflections between them. This approach offers two advantages: (1) the reduction of the experimental procedures to those required for the determination of the bidirectional transmittance and reflectance of fenestration layers, rather than complete fenestration systems, and (2) the determination of the net absorptance of each layer as part of the fenestration system, rather than the total absorptance of the complete fenestration system.

INTRODUCTION

A quantitative understanding of the solar-optical properties of fenestration systems is essential for accurate calculation of daylight illuminance levels, glare potential, solar heat gain, and thermal comfort. For clear, tinted, or reflective glass, for each direction of incoming radiation, there are only two specific directions of outgoing radiant flux: the direction of the transmitted radiation and the direction of the specularly reflected radiation. In this case, the solar-optical properties are expressed as simple functions of the incident angle of the incoming radiation. However, very little is known about the properties of fenestration systems that are optically more complex, such as systems that incorporate diffusive glass, venetian blinds, horizontal or vertical louvers, solar screens, etc. In this case, for each direction of incoming radiation, there is a particular 4π distribution of outgoing radiant flux, either transmitted or reflected by the fenestration system. For a complete description of the radiant behavior of such complex fenestration components, it is necessary to express their solar-optical properties as functions of both the incoming and the outgoing directions of the radiant flux.

For the purposes of determining such bidirectional properties, a scanning radiometer is under development at the Lawrence Berkeley Laboratory. This facility has been designed to measure the solar and visible bidirectional transmittance and reflectance of fenestration components and systems of arbitrary complexity (Spitzglas 1986). Determining the bidirectional properties of actual fenestration systems through direct measurement allows us to avoid the assumptions about the geometry and the texture of fenestration components that are commonly used in mathematical modeling. However, various combinations of even the most common fenestration components can produce thousands of optically different fenestration systems. Measuring all such combinations is practically impossible; moreover, the scanning radiometer provides no information about the net absorptance of individual layers as they perform as parts of fenestration systems. The layer-by-layer absorption of solar radiation, which ultimately contributes to solar heat gain through re-conduction, is a complicated function of the distribution of the incident radiation and the nature of the interreflections between the fenestration layers.

K. M. Papamichael, Research Associate, Windows and Daylighting Group, Applied Science Division, Lawrence Berkeley Laboratory; and F. C. Winkelmann, Staff Scientist, Simulation Research Group, Applied Science Division, Lawrence Berkeley Laboratory.

A mathematical procedure is therefore required to determine the overall optical properties of a fenestration system from the properties of each individual layer. This paper describes such a procedure. It is based on a matrix representation of the bidirectional properties of fenestration layers and systems. A computer program named TRA (Transmittance Reflectance Absorptance) was developed as an application of the method. The output of TRA serves as input to the daylighting calculation model SUPERLITE (Selkowitz et al. 1982; Modest 1982; Windows and Daylighting Group 1985) for determining daylight illuminance and luminance distributions and as input to heat transfer calculation models such as WINDOW-2.0 (Rubin et al. 1986) and DOE-2 (Building Energy Simulation Group 1984, 1985) for determining solar heat gain. This approach offers the capability to determine the hourly, seasonal, or annual luminous and thermal performance of fenestration systems of arbitrary complexity under varying environmental conditions in an accurate and consistent way.

BIDIRECTIONAL SOLAR-OPTICAL PROPERTIES

The bidirectional solar-optical properties of an optical element describe the fraction of the incoming radiant flux incident in direction (ζ_{in},θ_{in}) that leaves in each direction $(\zeta_{out},\theta_{out})$ (Figure 1). Fenestration systems usually define a plane that separates the environment into two hemispheres. If the incoming and the outgoing directions are in the same hemisphere, the solar-optical property is called *reflectance* ; otherwise, it is called *transmittance*. The fraction that is neither transmitted nor reflected is called *absorptance*. The solar-optical properties that describe the alteration of the incoming radiation by a fenestration system are then expressed by the following functions:

front transmittance	$= \mathrm{ft}\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$,	(1a)
front reflectance	$= \mathrm{fr}\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$,	(1b)
front absorptance	$= \mathrm{fa}\,(\zeta_{in},\theta_{in})$,	(1c)
back transmittance	$= \mathrm{bt}\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$,	(1d)
back reflectance	$= \mathrm{br}\,(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$, and	(1e)
back absorptance	$= \mathrm{ba}\,(\zeta_{in},\theta_{in})$,	(1f)

where "front" represents the outside hemisphere, referring to radiation coming from outdoors, and "back" represents the inside hemisphere, referring to radiation coming from indoors.

The bidirectional solar-optical properties can be represented in matrix form by dividing the incoming and outgoing hemispheres into small solid-angle elements centered around a discrete set of *m* incoming directions $((\zeta_{in},\theta_{in})_j,\ j=1,m)$ and a discrete set of *n* outgoing directions $((\zeta_{out},\theta_{out})_i,\ i=1,n)$. An arbitrary incoming direction can then be associated with the single index *j* and an outgoing direction with the single index *i*. The transmittance function $\mathrm{ft}(\zeta_{out},\theta_{out},\zeta_{in},\theta_{in})$, for example, can then be represented by the matrix:

$$\begin{vmatrix} \mathrm{ft}(1,1) & \mathrm{ft}(1,2) & \mathrm{ft}(1,3) & \dots & \mathrm{ft}(1,m) \\ \mathrm{ft}(2,1) & \mathrm{ft}(2,2) & \mathrm{ft}(2,3) & \dots & \mathrm{ft}(2,m) \\ \mathrm{ft}(3,1) & \mathrm{ft}(3,2) & \mathrm{ft}(3,3) & \dots & \mathrm{ft}(3,m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathrm{ft}(n,1) & \mathrm{ft}(n,2) & \mathrm{ft}(n,3) & \dots & \mathrm{ft}(n,m) \end{vmatrix} \qquad (2)$$

where $\mathrm{ft}(i,j) = \mathrm{ft}((\zeta_{out},\theta_{out})_i,\ (\zeta_{in},\theta_{in})_j)$.

The transmitted flux T(i) in direction *i* is obtained from the incident flux distribution (I(j), j=1,m) by the following summation:

$$T(i) = \sum_{j=1}^{m} \mathrm{ft}(i,j)\cdot I(j) \qquad \text{where } i=1,\dots,n \qquad (3)$$

The transmitted flux in the *n* outgoing directions can also be determined by the following expression, which is the equivalent of Equation 3 in matrix format:

$$\begin{vmatrix} T(1) \\ T(2) \\ T(3) \\ \vdots \\ T(n) \end{vmatrix} = \begin{vmatrix} ft(1,1) & ft(1,2) & ft(1,3) & \dots & ft(1,m) \\ ft(2,1) & ft(2,2) & ft(2,3) & \dots & ft(2,m) \\ ft(3,1) & ft(3,2) & ft(3,3) & \dots & ft(3,m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ft(n,1) & ft(n,2) & ft(n,3) & \dots & ft(n,m) \end{vmatrix} \cdot \begin{vmatrix} I(1) \\ I(2) \\ I(3) \\ \vdots \\ I(m) \end{vmatrix} \qquad (4)$$

or, using boldface for the matrices, this may be written in very compact form as:

$$\mathbf{T} = \mathbf{ft} \cdot \mathbf{I} \qquad (5)$$

Similarly, the reflected flux is given by $\mathbf{R}=\mathbf{fr}\cdot\mathbf{I}$ and the total absorbed radiation is $\mathbf{fa}\cdot\mathbf{I}$ where **fa** is the row vector:

$$\begin{vmatrix} fa(1) & fa(2) & fa(3) & \dots & fa(m) \end{vmatrix} \qquad (6)$$

We note that, of the six properties in Equations 1a through 1f, only **ft**, **fr**, and **br** need to be measured, since **bt** can be obtained from **ft** by optical reciprocity, and the absorptances **fa** and **ba** can be calculated from the corresponding transmittance and reflectance matrices.

MULTILAYER FENESTRATION SYSTEMS

The power of the matrix approach is evident when we apply it to determining the net bidirectional properties of a multilayer fenestration system when the transmittance and reflectance distributions for the constituent layers are known. We consider the outgoing flux from each layer to be an incoming flux to the next layer in sequence (Figure 2). For a two-layer system, let $\mathbf{ft_1}$ and $\mathbf{ft_2}$ be the front transmittance matrices for layers 1 and 2, respectively. For incident flux **I**, using Equation 5, the transmitted flux $\mathbf{T_1}$ through layer 1 is determined by:

$$\mathbf{T_1} = \mathbf{ft_1} \cdot \mathbf{I} \qquad (7)$$

Flux $\mathbf{T_1}$ is incident on layer 2, so, without interreflections, the transmitted flux $\mathbf{T_2}$ through layer 2 is determined by:

$$\mathbf{T_2} = \mathbf{ft_2} \cdot \mathbf{T_1} = \mathbf{ft_2} \cdot (\mathbf{ft_1} \cdot \mathbf{I}) \qquad (8)$$

or, using the standard associative property of matrix multiplication:

$$\mathbf{T_2} = (\mathbf{ft_2} \cdot \mathbf{ft_1}) \cdot \mathbf{I} = \mathbf{ft} \cdot \mathbf{I} \qquad (9)$$

where

$$\mathbf{ft} = \mathbf{ft_2} \cdot \mathbf{ft_1} \qquad (10)$$

The overall bidirectional transmittance of the two-layer system can thus be expressed in terms of the product of the transmittance matrices of the constituent layers.

We illustrate the details of the various matrix multiplications that take place for a two-layer system by considering the simple case of only two incoming and two outgoing directions (Figure 3). Let the incident flux be:

$$\mathbf{I} = \begin{vmatrix} I(1) \\ I(2) \end{vmatrix} \qquad (11)$$

The flux $\mathbf{T_1}$ transmitted into the gap between the two layers is $\mathbf{T_1}=\mathbf{ft_1}\cdot\mathbf{I}$, or, by expanding the matrices:

$$\begin{vmatrix} T_1(1) \\ T_1(2) \end{vmatrix} = \begin{vmatrix} ft_1(1,1) & ft_1(1,2) \\ ft_1(2,1) & ft_1(2,2) \end{vmatrix} \cdot \begin{vmatrix} I(1) \\ I(2) \end{vmatrix} = \begin{vmatrix} ft_1(1,1)\cdot I(1)+ft_1(1,2)\cdot I(2) \\ ft_1(2,1)\cdot I(1)+ft_1(2,2)\cdot I(2) \end{vmatrix} \qquad (12)$$

Thus $T_1(1)$, for example, consists of a fraction $ft_1(1,1)$ of $I(1)$, and a fraction $ft_1(1,2)$ of $I(2)$.† Neglecting interreflections, the flux $\mathbf{T_2}$ transmitted through the second layer is $\mathbf{T_2=ft_2 \cdot T_1}$, or, by expanding the matrices:

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1) & ft_2(1,2) \\ ft_2(2,1) & ft_2(2,2) \end{vmatrix} \cdot \begin{vmatrix} T_1(1) \\ T_1(2) \end{vmatrix} \tag{13a}$$

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1) & ft_2(1,2) \\ ft_2(2,1) & ft_2(2,2) \end{vmatrix} \cdot \begin{vmatrix} ft_1(1,1)\cdot I(1)+ft_1(1,2)\cdot I(2) \\ ft_1(2,1)\cdot I(1)+ft_1(2,2)\cdot I(2) \end{vmatrix} \tag{13b}$$

$$\begin{vmatrix} T_2(1) \\ T_2(2) \end{vmatrix} = \begin{vmatrix} ft_2(1,1)\cdot ft_1(1,1)\cdot I(1)+ft_2(1,1)\cdot ft_1(1,2)\cdot I(2)+ft_2(1,2)\cdot ft_1(2,1)\cdot I(1)+ft_2(1,2)\cdot ft_1(2,2)\cdot I(2) \\ ft_2(2,1)\cdot ft_1(1,1)\cdot I(1)+ft_2(2,1)\cdot ft_1(1,2)\cdot I(2)+ft_2(2,2)\cdot ft_1(2,1)\cdot I(1)+ft_2(2,2)\cdot ft_1(2,2)\cdot I(2) \end{vmatrix} \tag{13c}$$

Returning to the general case of an arbitrary number of incoming and outgoing directions, we consider Equation 9 with one interreflection between the two layers and we obtain (Figure 4):

$$\mathbf{ft = ft_2 \cdot ft_1 + ft_2 \cdot br_1 \cdot fr_2 \cdot ft_1 = ft_2 \cdot (1+br_1 \cdot fr_2) \cdot ft_1} \tag{14}$$

Including second- and higher-order interreflections, we obtain the infinite series:

$$\mathbf{ft = ft_2 \cdot (1 + (br_1 \cdot fr_2) + (br_1 \cdot fr_2)^2 + (br_1 \cdot fr_2)^3 + \ldots) \cdot ft_1} \tag{15}$$

which is equivalent to:

$$\mathbf{ft = ft_2 \cdot (1-br_1 \cdot fr_2)^{-1} \cdot ft_1} \tag{16}$$

where $\mathbf{(1-br_1 \cdot fr_2)^{-1}}$ is the matrix inverse of $\mathbf{(1-br_1 \cdot fr_2)}$.

The matrix expressions for the other bidirectional properties of a two-layer fenestration system are listed in Table 1. Expressions for three or more layers can be obtained by sequential application of the two-layer relationships. Spectrally, selective layers can be handled by carrying through the matrix analysis for each of several wavelength bands.

The matrix approach described will be validated, and limitations such as edge effects will be evaluated using scanning radiometer (Spitzglas 1986) data on single layers and their combinations.

CONCLUSIONS

For accurate determination of the luminous and thermal performance of a total fenestration system, it is necessary to know its bidirectional solar-optical properties. These properties can be directly calculated only for optically simple layers (primarily glazing). Bidirectional transmittance and reflectance can be determined experimentally for fenestration system of arbitrary complexity using a scanning radiometer, after which directional absorptance can be calculated. However, the scanning radiometer provides no information about the net absorptance of the layers of the fenestration system.

Representing bidirectional properties of fenestration layers using matrices is simple and convenient. Matrix operations can then be used to determine bidirectional solar-optical properties of multilayer systems, based on the properties of their layers. For determining the properties of multilayer systems, this approach has the advantages over experimental procedures of minimizing time and cost and, most important, providing additional information on the radiation absorbed by each layer of the system.

The computer program TRA, an application of the matrix representation approach, generates input for appropriate daylighting calculation models, e.g. SUPERLITE, and heat transfer calculation models, e.g., WINDOW 2.0 and DOE-2, to determine the hourly, seasonal, and annual luminous and thermal performance of fenestration systems of arbitrary complexity for varying environmental conditions. These capabilities for more accurate and consistent modeling of realistic fenestration systems should contribute to better comparisons and optimizations of fenestration systems for use in buildings.

† For ordinary clear, tinted, or reflective glass the matrix ft_1 would be diagonal, i.e., $ft_1(1,2)$ =m $ft_1(2,1) = 0$. This would result in $T_1(1) = ft_1(1,1) \cdot I(1)$ and $T_1(2) = ft_1(2,2) \cdot I(2)$.

REFERENCES

Building Energy Simulation Group. 1984. "Overview of the DOE-2 building energy analysis program." Lawrence Berkeley Laboratory, Report LBL-19735, UC-95d.

Building Energy Simulation Group. 1985. "DOE-2 supplement: Version 2.1C." Lawrence Berkeley Laboratory, Report LBL-8706, Rev. 4 Suppl.

Modest, M. F. 1982. "A general model for the calculation of daylighting in interior spaces." *Energy and Buildings*, Vol. 5, pp. 69-72.

Rubin, M.; Arasteh, D.; and Hartman, J. 1986. "WINDOW: A computer program for calculating U-values and shadingcoefficients of windows." Lawrence Berkeley Laboratory, Report LBL-20212 (to be published).

Selkowitz, S.; Kim, J. J.; Navvab, M.; and Winkelmann, F. C. 1982. "The DOE-2 and SUPERLITE daylighting programs." *Proceedings of the 7th National Passive Solar Conference*, Knoxville, TN.

Spitzglas, M. 1986. "A new bidirectional scanner for testing building fenestration systems." Lawrence Berkeley Laboratory, Report LBL-21353 (to be published).

Windows and Daylighting Group. 1985. "SUPERLITE 1.0: Evaluation manual." Lawrence Berkeley Laboratory, Report LBL-19320.

ACKNOWLEDGMENTS

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Building and Community Systems, Building Systems Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, with cosponsorship by the Electric Power Research Institute and the New York State Energy Research and Development Authority under Contract No. RP2418-5-1, under the management of the Lighting Research Institute, Contract No. 85:IMP:1.

TABLE 1

Bidirectional Properties of a Two-Layer Fenestration System

Expressed in Terms of Matrices Representing the Properties of the Individual Layers,

where $\mathbf{interf}=(\mathbf{1}-\mathbf{br_1}\cdot\mathbf{fr_2})^{-1}$ and $\mathbf{interb}=(\mathbf{1}-\mathbf{fr_2}\cdot\mathbf{br_1})^{-1}$.

Property of fenestration system	Mathematical expression
front transmittance of system	$\mathbf{ft} = \mathbf{ft_2}\cdot\mathbf{interf}\cdot\mathbf{ft_1}$
front reflectance of system	$\mathbf{fr} = \mathbf{fr_1} + \mathbf{bt_1}\cdot\mathbf{fr_2}\cdot\mathbf{interf}\cdot\mathbf{ft_1}$
total absorptance of layer 1 for front flux	$\mathbf{ta_1f} = \mathbf{fa_1} + \mathbf{ba_1}\cdot\mathbf{fr_2}\cdot\mathbf{interf}\cdot\mathbf{ft_1}$
total absorptance of layer 2 for front flux	$\mathbf{ta_2f} = \mathbf{fa_2}\cdot\mathbf{interf}\cdot\mathbf{ft_1}$
back transmittance of system	$\mathbf{bt} = \mathbf{bt_1}\cdot\mathbf{interb}\cdot\mathbf{bt_2}$
back reflectance of system	$\mathbf{br} = \mathbf{br_2} + \mathbf{ft_2}\cdot\mathbf{br_1}\cdot\mathbf{interb}\cdot\mathbf{bt_2}$
total absorptance of layer 1 for back flux	$\mathbf{ta_1b} = \mathbf{ba_1}\cdot\mathbf{interb}\cdot\mathbf{bt_2}$
total absorptance of layer 2 for back flux	$\mathbf{ta_2b} = \mathbf{ba_2} + \mathbf{fa_2}\cdot\mathbf{br_2}\cdot\mathbf{interb}\cdot\mathbf{bt_2}$

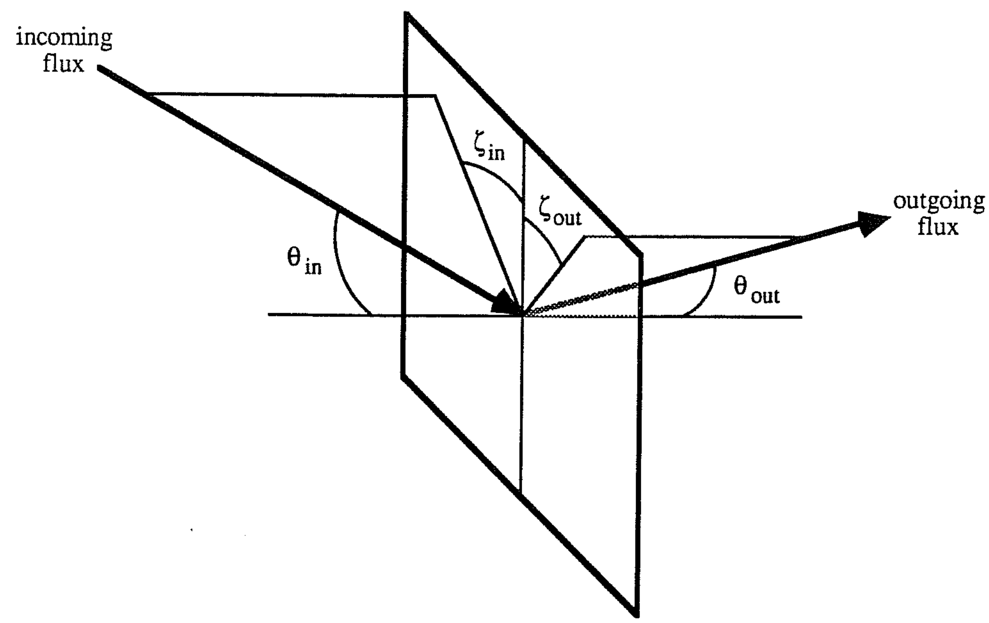

Figure 1. Schematic of an optical element showing the pairs of angles (ζ_{in}, θ_{in}) and (ζ_{out}, θ_{out}) that specify the directions of incoming and outgoing radiant flux.

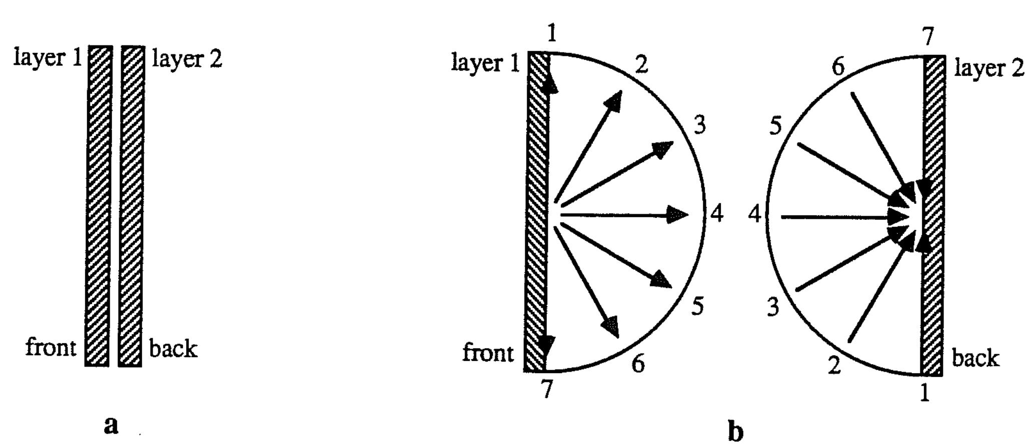

Figure 2. Schematic section of a two-layer fenestration system (a). The outgoing directional fluxes from one layer are considered to be incoming directional fluxes to the other layer (b).

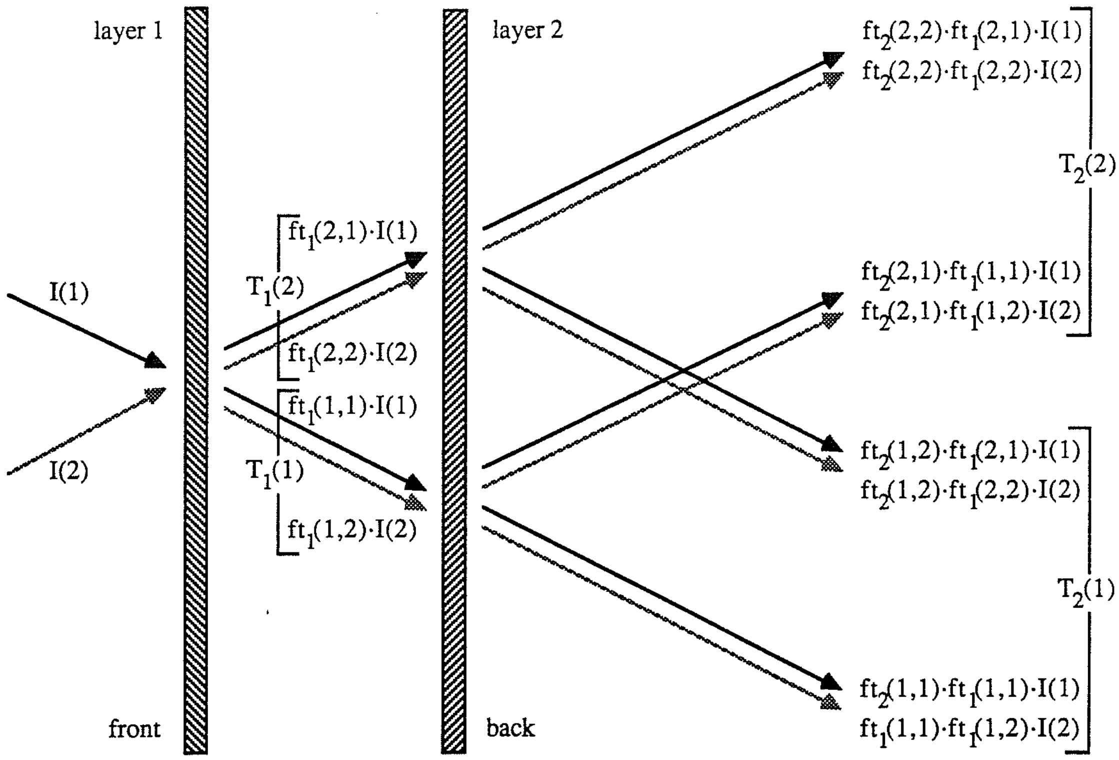

Figure 3. Schematic section of a two-layer fenestration system. A simplified case is illustrated, showing how the components of the incident flux propagate through the system. Two incoming and two outgoing directions are considered for each layer.

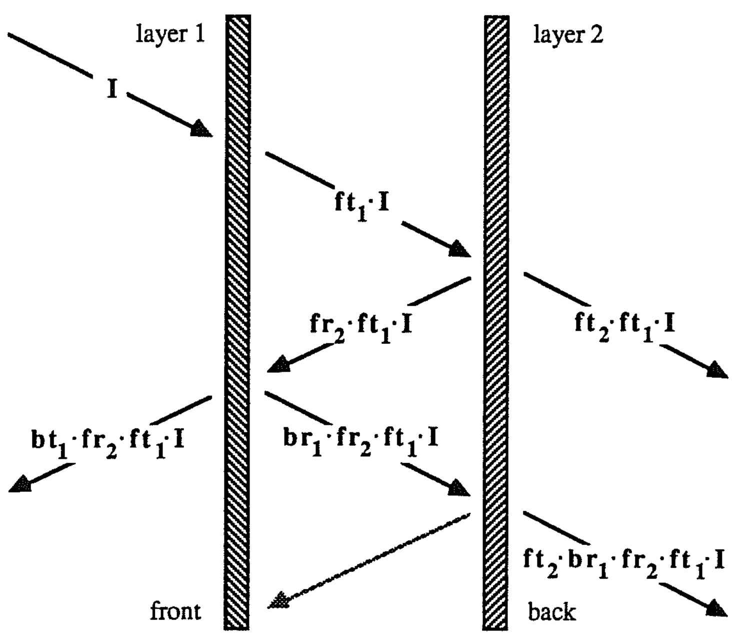

Figure 4. Schematic section of a two-layer fenestration system, showing the interreflections between the two layers, expressed as matrix multiplications. The matrix **I** represents the distribution of the incoming radiant flux.