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CALCULATING HEAT TRANSFER THROUGH WINDOWS

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SUMMARY

To calculate the energy performance of buildings, one must know the heat-transfer characteristics of the windows as functions of environmental variables, such as temperature and wind speed. Window designs are becoming more complex in response to the need for energy conservation. In this paper, we develop a general procedure for calculating the net energy flux through the glazed area of a window composed of an arbitrary number of solid layers. These layers, which may have thin-film coatings, can have any specified solar and thermal radiation properties and enclosed spaces between solid layers can contain either air or other gases. We verified our results by comparing them with experimental measurements of heat flow using a calibrated hot-box.

KEY WORDS Windows Heat transfer Energy conservation

INTRODUCTION

Because of the high cost and subtle performance differences of many new window designs, accurate evaluation of their potential for conserving energy is particularly important. To assess the relative merits of these windows, one needs to determine their energy performance on an annual basis in different climates and orientations. Of particular interest in this regard is the evaluation of the balance between minimizing heat loss and maximizing transmitted solar energy.

For our thermal model to be most useful, it must encompass both conventional single- and multiple-glazed windows and new designs, which may incorporate exotic materials and thin-film coatings. Advanced window products available on the market include: sun control films having a high solar reflectance, heat-mirror coatings and gas fills that add thermal insulation, antireflection coatings for increasing solar transmittance, and uncoated plastic films that replace conventional glass panes.

A commonly used procedure for calculating heat-transfer rates through windows is given by ASHRAE (1977). Typical of most other methods that use closed-form solutions, the ASHRAE method is limited to one or two glazing layers, whereas the energy-balance procedure of this paper is written for an arbitrary number of layers. Another deficiency of existing methods is that they can model only those materials that are completely opaque to infra-red radiation. Partially transmitting materials, such as plastic films, or any type of surface coating can be treated with the radiation-balance method developed in this paper. We have modified the equations commonly used for calculating heat-transfer rates due to conduction and convection in enclosed air spaces to apply to gases with lower thermal conductivities than air.

For calculating heating and cooling loads, the single value of interest is the net heat flux into the room. The calculation procedure discussed in this paper can also be used to determine the heat fluxes between the various layers of the window. Knowing these auxiliary fluxes is useful for isolating and understanding the physical mechanisms of heat transfer in windows. The temperature distribution across the window is another valuable by-product of this calculation, particularly for studies of condensation problems or thermal comfort.

In the first few sections of this paper, we discuss the thermal and radiative properties of glazing materials, followed by a detailed description of the radiation balance, including a method for dealing with thermal radiation from the atmosphere. Next, we present a series of equations for calculating heat transfer by conduction and convection in the enclosed air spaces and at the inside and outside surfaces

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of the window. We then set up the complete steady-state energy balance for the window and solve for the heat fluxes; because of the dependence of the various heat-transfer coefficients on the unknown temperatures of the layers, these equations must be solved iteratively. Finally, sample calculations of heat flux are presented for windows with up to four layers, including some with infra-red-transparent plastic films, together with results of experimental testing using a calibrated hot-box to validate our findings.

CONDUCTION IN SOLID LAYERS

Air, when motionless, is a good insulator. Most window materials have a small, if not negligible, resistance to conductive heat flow compared to the surrounding air layers. This characteristic is the rationale for multiple-glazed windows, which have 'dead' air spaces between the panes. Even single glazing derives most of its insulating value from the slowly moving air films near the indoor and outdoor surfaces. Single glazing has an overall conductance, or U-value, of about $6\ \mathrm{W m^{-2} K^{-1}}$, which is much less than the conductance of glass sheets or thin plastic materials (see Table I).

SOLAR OPTICAL PROPERTIES

Solar heat gain through a window system can be determined from the overall transmittance and the absorptance of each pane or layer as a function of the angle of incidence. The transmittance value is required to determine the fraction of direct solar radiation entering the room for any given angle of incidence, which changes with the sun's motion during the day. A hemispherical average transmittance value is needed to account for diffuse sky radiation and radiation reflected from the ground. The absorptance is needed because some of the short-wave solar radiation absorbed by the window is reradiated and convected to the room. In a window composed of layers separated by large thermal resistances, e.g. airspaces, the absorptance must be known for each layer, since a different fraction of the absorbed solar energy will be transferred inwards depending on the resistances between layers and their respective temperatures. All properties are averaged over wavelength and weighted by the solar spectral irradiance at sea-level. These properties can be calculated either from the optical indices of the window materials or from the measured properties of the individual layers (Rubin, 1982).

THERMAL RADIATION

In the thermal infra-red spectrum, as in the solar spectrum, the wavelength-averaged radiation properties of the glazing materials must be known to calculate the net radiation balance of a window. This integrated average is weighted by the emissive power of a blackbody at ambient temperature (the result of the average will not depend strongly on the choice of source temperature). Even though the surfaces may be characterized as specularly reflecting in the infra-red, only the hemispherical-average infra-red properties are required to calculate the heat flux, because all of the sources of thermal radiation are diffusely emitting. Furthermore, for the special geometry of parallel planes with high aspect ratios, the radiative interchange does not depend on the nature of the surfaces.

Table I. Conduction of heat in glazing materials.

glazing material	thermal conductivity $\frac{\mathrm{W}}{\mathrm{mK}}$	typical thickness m	thermal conductance $\frac{\mathrm{W}}{\mathrm{m^2K}}$
glass	0·9	0·003	300
polyester film	0·14	0·0001	1400
acrylic or polycarbonate sheet	0·19	0·006	30

In the case of opaque materials, we need only the reflectivity or emissivity of the two surfaces, but for partially transparent materials, such as some plastic films less than about 0·3 mm thick, both the reflectance and transmittance are required. Transmittance measured from either side will always be the same but, for an asymmetric material such as a thin-film coated pane, the reflectance values will generally differ from one side to the other.

Each layer acts as a passive element characterized by its transmission, reflection and absorption coefficients. In addition, each layer is a source of radiation so that, in contrast to the solar calculation, the energy-balance equations for the infra-red radiation form an inhomogeneous linear system.

Consider a system composed of N solid layers, or $2N$ surfaces, numbered consecutively from the outdoor layer or outdoor facing-surface (Figure 1). If infra-red-absorbing gases are not allowed in the gaps between layers, no energy is lost in transit between layers. We define the net infra-red energy flux leaving the kth surface as Q^{r}_k, where the superscript r denotes radiation. Then, for the nth layer (which is bounded by surfaces $2n-1$ and $2n$) we have

$$Q^{\mathrm{r}}_{2n} = S_{2n} + R_{2n}Q^{\mathrm{r}}_{2n+1} + T_nQ^{\mathrm{r}}_{2n-2} \tag{1}$$

$$Q^{\mathrm{r}}_{2n-1} = S^{\mathrm{r}}_{2n-1} + R_{2n-1}Q^{\mathrm{r}}_{2n-2} + T_nQ^{\mathrm{r}}_{2n+1} \tag{2}$$

where R_k is the infra-red reflectance of the layer measured from the kth surface, and T_n is the transmittance (which is the same from either side) of the nth layer. The emitted energy flux from the kth surface, S_k, is given by

$$S_k = \epsilon_k\sigma\theta_n^4 \tag{3}$$

where ϵ_k is the emissivity of the kth surface, θ_n is the temperature of layer n, and k is $2n$ or $2n-1$. Because two surfaces separated by a small thermal resistance will have nearly the same temperature, we may assign a single temperature to layer n. Glass panes or plastic films can certainly be treated as having a uniform temperature since even the first-order effect of their thermal resistance is negligible. Even for thick plastic sheets, this approximation will introduce an error of only a few per cent if the additional thermal resistance is included at the end of this calculation.

For layers 1 and N, the terms Q^{r}_0 and Q^{r}_{2N+1} appear in equations (1) and (2). These terms represent the infra-red sources from the outdoor environment and the room, which we will rename $Q^{\mathrm{r}}_{\mathrm{out}}$ and $Q^{\mathrm{r}}_{\mathrm{in}}$, respectively. For the room, $Q^{\mathrm{r}}_{\mathrm{in}} = \epsilon_{\mathrm{in}}\sigma\theta^4_{\mathrm{in}}$, where θ_{in} is taken to be the indoor air temperature and $\epsilon_{\mathrm{in}} = 1$, since the room enclosure is assumed to look like a black body to the aperture of the window. Caution

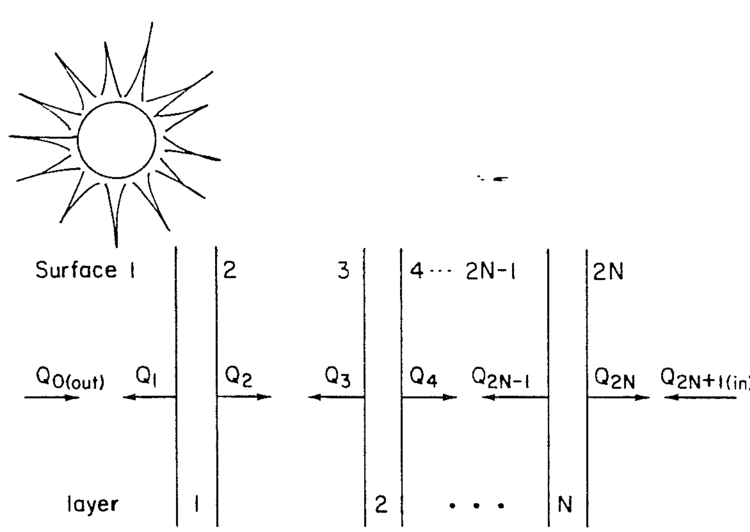

Figure 1. Net radiation balance for a window of N solid layers. Q_k is the energy flux from surface k. Q_{out} and Q_{in} are the energy fluxes from the environment and the room, respectively

should be exercised when making these assumptions about a room in a real building; since direct sunlight on an interior wall may raise the wall temperature well above air temperature, or large areas of exterior glazing or uninsulated wall may be nearly at the outside temperature. If the geometry of the room is such that the window in question receives a large fraction of the radiation emitted by these areas, then a detailed radiation balance must be performed to determine the net flux.

On overcast days, Q^{r}_{out} is a similar function of the outside air temperature, θ_{out}; however, clear skies have a lower emissive power than cloudy skies because of the transparency of the atmosphere. Swinbank (1963) reports the following correlation between the radiation falling on a horizontal surface, S_{sky} in W/m^2, and the dry-bulb temperature, θ_{out} in K,

$$S_{\text{sky}} = 5{\cdot}31 \times 10^{-13}\, \theta_{\text{out}}^{6} \tag{4}$$

Equation (4) is found to be independent of location and, based on theoretical considerations, is expected to be independent of altitude. We define a sky emittance as

$$\epsilon_{\text{sky}} = \frac{S_{\text{sky}}}{\sigma \theta_{\text{out}}^{4}} = 9{\cdot}27 \times 10^{-6}\, \theta_{\text{out}}^{2} \tag{5}$$

Then the effective outside emittance, ϵ_{out}, is just the weighted average of ϵ_{sky} and the emittance of the rest of the environment (ground, obstructions, and cloudy sky), which we take to be one. If F_{sky} is the view factor of the sky from the window ($F_{\text{sky}} = 1$ for a horizontal skylight or 1/2 for an unobstructed vertical window), and F_{c} is the fraction of sky which is clear,

$$\epsilon_{\text{out}} = F_{\text{sky}} F_{\text{c}} \epsilon_{\text{sky}} + (1 - F_{\text{sky}} F_{\text{c}}) \tag{6}$$

F_{c} can be obtained from standard meteorological reports of fractional cloud cover.

All of the terms in the N pairs of equations (1) and (2) have now been defined. These equations form an inhomogeneous linear set, which we arrange in the matrix form

$$\sum_{j} M_{ij} Q^{r}_{j} = S_i \tag{7}$$

The $2N \times 2N$ matrix $[\mathbf{M}]$ has the structure

$$M_{ii} = 1 \tag{8}$$

$$M_{2n-1,2n-2} = -R_{2n-1} \tag{9}$$

$$M_{2n-1,2n+1} = -T_n \tag{10}$$

$$M_{2n,2n-2} = -T_n \tag{11}$$

$$M_{2n,2n+1} = -R_{2n} \tag{12}$$

All other elements are zero. Given the temperatures of each layer, one can compute the sources, S_k, and, therefore, the net fluxes by

$$Q_i = \sum_{j} (M^{-1})_{ij} S_j \tag{13}$$

NATURAL CONVECTION/CONDUCTION IN GAPS

A 'conductance', h is defined such that the net heat flux due to conduction and convection between layers n and $n+1$ is given by

$$Q = h(\theta_n - \theta_{n+1}) \tag{14}$$

We divide the heat flux somewhat artificially into two parts to parallel the convention used for the infra-red fluxes.

$$Q^{c}_{2n} = h\theta_n \tag{15}$$

$$Q^{c}_{2n+1} = h\theta_{n+1} \tag{16}$$

The conductance is given by

$$h = \frac{\lambda}{w} Nu \tag{17}$$

where λ is the thermal conductivity of the gas in the gap, w is the width of the gap, and Nu is the Nusselt number. For vertical air layers, the Nusselt number is given by De Graff and Van der Held (1953) as

$$Nu = \begin{cases} 1 & Gr < 7\times 10^3 \\ 0{\cdot}0384\, Gr^{0{\cdot}37} & 10^4 < Gr < 8\times 10^4 \\ 0{\cdot}0317\, Gr^{0{\cdot}37} & Gr > 2\times 10^5 \end{cases} \tag{18}$$

Three flow regimes are distinguished: pure conduction, transition, and boundary layer. The type of flow is determined by the Grashoff number

$$Gr_w = \frac{g\beta\rho^2 w^3}{\mu^2} \Delta\theta \tag{19}$$

where g is the gravitational acceleration ($9{\cdot}8$ m/s^2), β is the coefficient of thermal expansion, ρ is the density, μ is the viscosity, and $\Delta\theta$ is the temperature difference across the region. Discontinuities exist in equation (18); examination of De Graff's data indicates that the expression for the transition region can be extended without significant error down to $Gr_w = 7\times 10^3$. In the region between the defined transition and boundary layer regimes, we fit the following expression:

$$Nu = 0{\cdot}41\, Gr^{0.16} \qquad 8\times 10^4 < Gr < 2\times 10^5 \tag{20}$$

For inclined or horizontal surfaces, De Graff gives other expressions for Nu.

Nu, for any gas, will be a function only of the product $Gr_w Pr$ (sometimes called the Rayleigh number, Ra). The Prandtl number, Pr, is the dimensionless ratio, $c_p\mu/\lambda$, where c_p is the specific heat at constant pressure. De Graff eliminated the explicit dependence on Pr from equation (18) on the assumption that, under ambient conditions, Pr is a constant for air. We introduce Pr to generalize the procedure for any gas-fill:

$$Nu = \begin{cases} 1 & GrPr < 5\times 10^3 \\ 0{\cdot}0429\,(GrPr)^{0{\cdot}37} & 5\times 10^3 < GrPr < 6\times 10^4 \\ 0{\cdot}43\,(GrPr)^{0{\cdot}16} & 6\times 10^4 \leq GrPr < 1{\cdot}5\times 10^5 \\ 0{\cdot}0354\,(GrPr)^{0{\cdot}37} & GrPr > 1{\cdot}5\times 10^5 \end{cases} \tag{21}$$

Table II gives some of the physical properties of gases that might be used in an architectural window. The variation in these properties over the range of ambient temperatures should be considered when doing detailed calculations. All of the gases listed have a lower thermal conductivity than air, which means that they will perform better at small gap spacings where conduction is the dominant mode of heat transfer. Low conductivity is a necessary but not sufficient condition for their use; cost, chemical reactions with other window components and toxicity must also be considered.

When conditions are such that heat transfer through a given gas mixture is predominantly by conduction, the effective thermal conductivity of the mixture is given by

$$\bar{\lambda} = \frac{\sum_i x_i\lambda_i M_i^{1/3}}{\sum_i x_i M_i^{1/3}} \tag{22}$$

Table II. Physical properties of gases at 0°C and atmospheric pressure

Gas	M ($\frac{g}{mole}$)	ρ ($\frac{kg}{m^3}$)	λ ($\frac{10^{-2}\,W}{m\,K}$)	μ ($\frac{10^{-5}\,kg}{m\,s}$)	c_p ($\frac{10^3\,J}{kg\,K}$)	β ($10^{-3}\,K^{-1}$)
Air	28·96	1·29	2·50	1·86	1·005	3·67
Ar	39·95	1·70	1·78	1·21	2·16	3·68
CO_2	44·01	1·98	1·60	1·42	0·828	3·72
SO_2	64·06	2·93	0·93	1·21	0·594	3·90
Kr	83·70	3·74	0·91	2·41	0·38	3·68
CCl_2F_2	120·91	5·4	0·90	1·05	0·558	
SF_6	146·05	6·60	1·30	1·46	0·616	

where x_i, M_i, and λ_i are the mole fraction, the molar mass, and the thermal conductivity of the ith component, respectively (Glaser, 1977). Calculation of radiation exchange through CO_2, SO_2, CCl_2F_2, and SF_6 is complicated because these gases have relatively strong absorption bands in the infra-red spectrum.

OUTSIDE SURFACE FILM COEFFICIENT

Forced convection due to windy conditions at the outdoor surface of the window was studied by Ito and Kimura (1972). Results from these experiments are used by Lokmanhekim (1975) to calculate the outdoor convection coefficient, h_{out}, in units of $Wm^{-2}\,K^{-1}$, which is found to be a function of wind speed, v, in m/s, and the direction of the wind with respect to the azimuth of the window, ϕ. On the windward side of the building, $(|\phi| < \pi/2)$,

$$h_{out} = \begin{cases} 8{\cdot}07\, v^{0{\cdot}605} & v > 2\ \mathrm{m/s} \\ 12{\cdot}27 & v < 2\ \mathrm{m/s} \end{cases} \tag{23}$$

On the leeward side, $(|\phi| \geqslant \pi/2)$,

$$h_{out} = 18{\cdot}64(0{\cdot}3 + 0{\cdot}05v)^{0{\cdot}605} \tag{24}$$

INDOOR SURFACE FILM COEFFICIENT

For the heat-transfer coefficient of vertical plates in air, in units of $Wm^{-2}\,K^{-1}$, McAdams (1954) gives the following expressions and these equations can be used to calculate free-convection heat transfer at the inside surface of the window:

$$h_{in} = \begin{cases} 1{\cdot}42\,(\Delta\theta/z)^{1/4} & 10^4 < Gr_z Pr < 10^9 \quad \text{laminar} \\ 1{\cdot}31\,\Delta\theta^{1/3} & 10^9 < Gr_z Pr < 10^{12} \quad \text{turbulent} \end{cases} \tag{25}$$

where z is the height of the surface in meters, and Gr_z depends on the height of the window and the temperature difference between the inside glass surface and room air in degrees Kelvin. The average of these temperatures should be used when specifying the thermophysical properties of air for calculating Gr.

THE STEADY-STATE ENERGY BALANCE

Under the assumption that environmental conditions (wind speed, solar position and intensity, inside and outside air temperautre, etc.) remain constant over the typical response time of the window, the temperature of each layer will be determined by the condition that no net energy is absorbed (or released) by any layer. This condition uniquely determines the temperature of each layer; we write these N non-linear equations in the N unknown temperatures as

$$\Delta_k(\{\theta_i, i=1, N\})=0 \tag{26}$$

The functions Δ_k are given by

$$\Delta_n = Q^{r}_{2n-1} + Q^{r}_{2n} - Q^{r}_{2n-2} - Q^{r}_{2n+1} + Q^{c}_{2n-1} + Q^{c}_{2n} - Q^{c}_{2n-2} - Q^{c}_{2n+1} - A_n I \tag{27}$$

I is the solar intensity outside the window and A_n is the fraction of incident solar energy absorbed in layer n.

The calculation is initialized by guessing a linear progression of the temperatures from the outside to the inside. The exact temperatures can be written as

$$\theta_k = \theta^0_k + \delta\theta_k \tag{28}$$

For $\delta\theta_k$ small, we expand to first order

$$\begin{aligned} \Delta_i(\{\theta_k\}) &= \Delta_i(\{\theta^0_k + \delta\theta_k\}) = 0 \\ &\approx \Delta_i(\{\theta^0_k\}) + \sum_j \left[\frac{\partial \Delta_i}{\partial \theta_j}\right] \delta\theta_j \end{aligned} \tag{29}$$

This equation can be solved for the $\delta\theta_k$ by inverting the matrix $[\partial\mathbf{\Delta}/\partial\boldsymbol{\theta}]$.

$$\delta\theta^1_i = -\sum_j \left[\frac{\partial \Delta}{\partial \theta}\right]^{-1}_{ij} \Delta_j(\{\theta^0_k\}) \tag{30}$$

If our guess was close, then a better approximation to the temperature distribution is

$$\theta^1_k = \theta^0_k + \delta\theta^1_k \tag{31}$$

The above procedure is repeated, beginning with equation (29), until the components of the solution converge to the desired accuracy.

Once the temperatures are known, the fluxes can be determined from equations (1), (2), (15) and (16). For example, the quantity of primary interest for calculating heating and cooling loads is the net heat flux into the room, or,

$$Q_{\text{net}} = Q^{r}_{2N} - Q^{r}_{\text{in}} + Q^{c}_{2N} - Q^{c}_{\text{in}} + T_{\text{sol}} I \tag{32}$$

where T_{sol} is the solar transmittance.

RESULTS AND DISCUSSION

Heat-transfer rates predicted by our model were found to be in good agreement with experimental results for double-pane glass windows with infra-red transparent polyester films suspended between the panes (see Figure 2). These rates are given in the form of overall thermal conductance or U-value, which is defined as

$$U = \frac{Q_{\text{net}}}{\theta_{\text{out}} - \theta_{\text{in}}} \tag{33}$$

A calibrated hot-box was used to measure the U-values under still air conditions and the results were corrected for a 6·7 m/s (15 mph) wind speed (Klems, 1979).

The U-value provides a rough measure of the response of a window to the difference in indoor and outdoor temperatures. For windows with at least two panes, U does not depend strongly on temperature or wind speed because of the buffering effect of the enclosed air spaces. A single-pane window with a

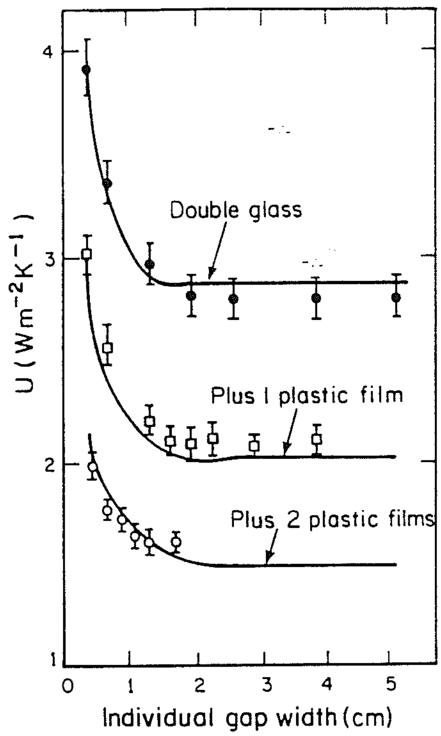

Figure 2. Overall thermal conductance measured in a calibrated hot-box vs. individual gap widths for double glazing alone, and double glazing with one or two polyester films suspended vertically in the air gap. The solid lines represent calculated values

low-emissivity coating, however, may have a conductance which varies by as much as 50 per cent over the range of typical outdoor conditions.

The shading coefficient is defined by ASHRAE (1977) as the ratio of the solar heat gain of the glazing under consideration to the solar heat gain through standard 1/8-inch glass. Here, solar heat gain is given by Q_{net} with $\theta_{out} = \theta_{in}$. The most serious problem with the shading coefficient concept is its very strong dependence on the angle of incidence for complex glazing systems. This effect is most pronounced when the functional form of the optical properties is very different from the properties of the reference glazing. An extreme example of this effect is in shading devices such as venetian blinds or louvred screens whose optical properties have a sharp discontinuity.

Both U-value and shading coefficient are useful for making comparisons between windows and, subject to the limitations outlined above, they can be used in simplified calculations of building energy consumption. In general, it is best to evaluate the energy flux in response to the continuously changing environmental conditions. Q_{net} can be calculated as part of a detailed building model that uses real weather data sampled hourly over a period of a month, a season, or a year. A simpler building model and monthly averaged weather data have been used in previous studies to find winter season heating requirements attributable to windows incorporating heat-mirror coatings as a function of orientation and climate (Rubin, Creswick and Selkowitz, 1980).

CONCLUSIONS

The agreement between observed and calculated results establishes the suitability of our techniques for modelling the various heat-transfer mechanisms in a window, and the accuracy of our computational methods. This model can be used to predict the thermal performance of conventional windows having advanced energy-conserving features. Furthermore, within the overall framework of this model, we can substitute appropriate equations for the heat-transfer coefficients of special components, such as those with rough or uneven surfaces, with no loss of generality.

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